# Noise Performance of Orthogonal RF Beamforming for Millimetre Wave Massive MIMO Communication Systems

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***Abstract*—Millimeter wave (mmwave) bands offer enormous untapped spectrum for broadband radio communications. For high dimensional and sparse multiple input multiple output (MIMO) channels, analog beamforming (ABF) and digital multi-stream beamforming (DBF), collectively known as hybrid beamforming (HBF), enable low cost and low power-consumption radio architectures with near-optimal performance. It is easier and more efficient to learn such channels in beamspace than in spatial signal space. For radio frequency (RF) beam training, the Tx-Rx beam combination yielding maximum receiver output is selected. Noise can cause false beam selections manifesting in communication rate loss. In this paper, analytically derived closed-form expressions and simulation results for such noise performance evaluation have been presented.**

***Index Terms*—Massive MIMO, sparse mmwave channel, beamspace MIMO, hybrid beamforming, noisy RF beam training, communication rate loss.**

## I. Introduction

Demand for wireless broadband data services has been growing very fast, enabling better human life and providing increased economic potential. Multi-antenna techniques, channel coding, interference co-ordination, etc. have been successfully employed to achieve optimum area spectral efficiency at conventional sub-6 GHz bands [1]. To meet increased demands for higher data rates, mmwave spectrum is being explored for providing wireless access to high data rate (gigabits per second) communications [2], [3]. Short-range mmwave communication specifications such as IEEE 802.11ad [5], IEEE 802.15.3c [6], and Wireless HD [7] have already been defined for wireless local area network (WLAN), wireless personal area network (WPAN), and wireless video area network (WVAN) applications, respectively. Due to very small wavelengths of mmwave frequencies, large numbers of antenna elements can be employed in small physical apertures. This leads to very high dimensional spatial MIMO channels. Such large arrays have very high array gain and extremely narrow beams. A few scatterers, typically, may exist in such small fields-of-view. Broadband mmwave systems encounter raised noise floors since thermal noise is directly proportional to bandwidth.

Further, due to the propagation characteristics of mmwave radio waves e.g. higher free space path loss owing to extremely high carrier frequencies, higher atmospheric absorption, foliage attenuation, etc., only a few multi-path components typically exist for communication [8], [9], [10]. In other words, the mmwave massive MIMO channel is, typically, rank-deficient or sparse in beamspace [8], [9], [10]. The HBF architecture was proposed in [11] to provide low cost, low power-consumption radio systems for mmwave broadband communications. The carbon footprint of information and communication technology (ICT) systems was as large as that of global air travel in 2008 [12]. The deployment of ICT systems has been growing since then and is likely to grow further in coming years. Therefore, the HBF architecture of [11] is very relevant for mmwave cellular systems.

The HBF [11] is implemented in two stages: 1) ABF stage during which the transmitter and receiver jointly design RF beamformers for maximizing the useful signal power for each data stream, ignoring the inter-stream interference, 2) zero-forcing (ZF) based DBF stage relying on the quantized channel vector estimate fed-back from the receiver, to manage, or ideally nullify in the case of well-conditioned channels, the inter-stream interference [11]. Using the concept of beamspace MIMO [9], [10], an orthogonal RF beamforming codebook delivers good performance for the first stage of the HBF algorithm, especially when the multi-path components are orthogonal to each other in which case the baseband digital precoder and combiner can be identity matrices as there will not be any inter-stream interference. Multi-level beam selection [13],

[14] is the state-of-the-art algorithm for RF beam training as compared to an exhaustive beam search since it offers significant savings in training overheads. Analog beam training is carried out in presence of noise. In the high SNR regime, when the noise variance is negligible as compared to the signal power, noise will have negligible impact on the selection of the transmit and receive RF beamforming vectors from the respective unitary codebooks. However, when the SNR is low i.e. the received signal and noise are of comparable power, then it is possible that noise leads to the selection of incorrect beamforming vectors causing very low communication rate for orthogonal RF beamforming codebooks.

This paper presents the analytical derivations for the probability of correct beam training in presence of independent and identically distributed (i.i.d.) additive white Gaussian noise (AWGN), mean communication rates and mean communication losses on account of false beam selections for exhaustive and multi-level RF beam search techniques using unitary beamforming codebooks.

The rest of the paper is organised as follows: Section II covers the mmwave cellular communication system architecture, Section III specifies the problem statement, Section IV presents analytical derivations of closed-form expressions for the noise performance of the RF beam training, Section V presents the simulation results and Section VI summarises and concludes the paper.

The following notations have been used: $\boldsymbol{X}$ is a matrix, $\boldsymbol{x}$ is a vector, $x$ is scalar, $\boldsymbol{I}_W$ is the identity matrix of dimensions $W \times W$, $\boldsymbol{C}^N$ is $N$ dimensional complex space, $\sim$ denotes "distributed as", $\boldsymbol{x} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{X})$ is a complex Gaussian random vector with zero vector $\boldsymbol{0}$ as its mean and $\boldsymbol{X}$ as its covariance matrix, $\delta_{m,l}$ is a discrete-sequence unit impulse which is equal to 1 only when $m = l$ and equal to zero otherwise, $E[.]$ is expectation operation, $\|\boldsymbol{X}\|_F$ is the Frobenius norm of matrix $\boldsymbol{X}$, $\|\boldsymbol{x}\|_2$ is 2-norm of vector $\boldsymbol{x}$, $|x|$ is magnitude of scalar $x$, $[\cdot]^*$ is conjugate transpose, and $[\cdot]^T$ is transpose.

## II. System Model

We consider the mmwave system architecture shown in Figure 1 [15]. It represents the downlink multi-user scenario in which the base station (BS) transmitter with $N_{TX}$ antennas and $N_{RF}$ radio frequency chains (RFCs) needs to communicate with $U$ mobile stations (MSs). Due to the sparsity of mmwave MIMO channels, we typically expect availability of two strong multi-path components (MPCs) per user. Without the loss of generality, we assume each MS to have $N_{RX}$ antennas with two RFCs since it allows two stream spatial multiplexing. Hence, the BS transmits $N_s = 2 \times U$ data symbol streams for $U$ users. $N_s = M \times U$ for $M$ streams per user, if the MPCs are available. Due to the larger resources at the BS and the sparsity, $N_{TX} \gg N_{RX} \gg M$, respectively. Since the maximum number of simultaneously transmitted streams are limited by the number of RFCs available at the BS transmitter $N_{RF}$, $N_s \leq N_{RF}$. The $N_s \times 1$ data symbol vector can be represented as in equation (1) such that transmit power constraint given by equation (2) is satisfied.

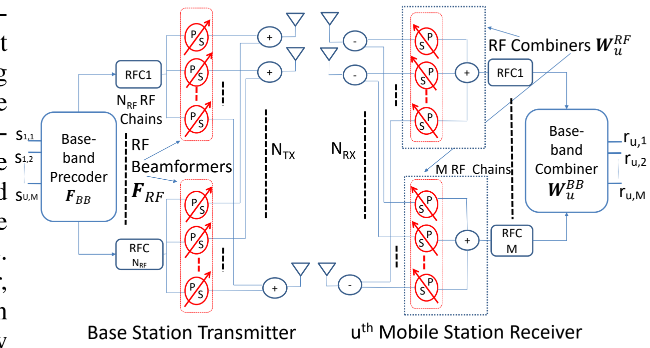


Figure 1. System model: BS serving $U$ MSs with transmission of $M$ streams per MS [15].

$$\boldsymbol{s} = [s_{1,1}, s_{1,2}, ..., s_{1,M}, s_{2,1}, s_{2,2}, ...s_{U,M}]^T \quad (1)$$

$$E[\boldsymbol{s}\boldsymbol{s}^*] = \frac{P}{N_s}\boldsymbol{I}_{N_s}, \quad (2)$$

where $P$ is the total power transmitted by the BS for communicating $N_s$ data-streams to $U$ users. $E[.]$ denotes the expectation operation and $^*$ denotes the conjugate transpose. The transmit power constraint equation (2) assumes that the transmitter allocates power equally to all the data-streams for ideal effective MIMO channel with unity condition number and the data symbol streams are uncorrelated with each other. These assumptions lead to maximum sum communication rate. Baseband precoder $\boldsymbol{F}_{BB}$ of dimensions $N_S \times N_S$ is modelled as in equation (3),

$$\boldsymbol{F}_{BB} = [\boldsymbol{f}_{1,1}^{BB}, \boldsymbol{f}_{1,2}^{BB}, ..., \boldsymbol{f}_{1,M}^{BB}, \boldsymbol{f}_{2,1}^{BB}, \boldsymbol{f}_{2,2}^{BB}, ...\boldsymbol{f}_{U,M}^{BB}]. \quad (3)$$

RF precoder of dimensions $N_{TX} \times N_S$ is given as in equation (4),

$$\boldsymbol{F}_{RF} = [\boldsymbol{f}_{1,1}^{RF}, \boldsymbol{f}_{1,2}^{RF}, ..., \boldsymbol{f}_{1,M}^{RF}, \boldsymbol{f}_{2,1}^{RF}, \boldsymbol{f}_{2,2}^{RF}, ...\boldsymbol{f}_{U,M}^{RF}]. \quad (4)$$

This work considers orthogonal RF beamforming codebooks which need only phase-shifts for beam steering and the aperture distribution is not changed [16]. This facilitates realisations using analog RF phase shifters with a constant modulus constraint as amplitude-control is not needed. Orthogonal beamforming codebooks have optimum performance for the first stage implementation for sparse mmwave Massive MIMO channels [?], [8], [9], [10] and have the advantage that they can be specified by a single parameter viz. the steering angle [11]. For simplicity, the uniform linear array (ULA) has been considered at both BS and MS. For steering the beam main response axis (MRA) to the direction $\theta$, the RF beamforming vector is given by equation (5),

$$\boldsymbol{f}^{RF}(\theta)=[1, e^{-j\frac{2\pi}{\lambda}dsin(\theta)}, ..., e^{-j(N-1)\frac{2\pi}{\lambda}dsin(\theta)}]/\sqrt{N}. \quad (5)$$

The entries of $\boldsymbol{F}_{RF}$ are normalised such that $|[\boldsymbol{F}_{RF}]_{m,n}|^2 = N_{TX}^{-1}$, where $[\boldsymbol{F}_{RF}]_{m,n}$ indicates entry corresponding to the $m^{th}$ row and $n^{th}$ column of the RF precoder matrix $\boldsymbol{F}_{RF}$ [11]. $N$ orthogonal beams can be formed with an $N$-element ULA. The MRAs in u-space corresponding to these $N$ orthogonal beams are obtained by equations (6) and (7) for even and odd values of $N$ respectively, where $u_n = \sin(\theta_n)$.

$$u_n = -1 + n\frac{2}{N}, \ n = 0, 1, ..., N-1 \quad (6)$$

$$u_n = -1 + \frac{1}{N} + n\frac{2}{N}, \ n = 0, 1, ..., N-1 \quad (7)$$

For orthogonal beamforming with ULAs, the transmit and receive RF beamforming codebooks $\boldsymbol{\mathcal{F}}$ and $\boldsymbol{\mathcal{W}}$ are unitary matrices as in equations (8) and (9), respectively.

$$\boldsymbol{\mathcal{F}}\boldsymbol{\mathcal{F}}^* = \boldsymbol{\mathcal{F}}^*\boldsymbol{\mathcal{F}} = \boldsymbol{I}_{N_{TX}} \quad (8)$$

$$\boldsymbol{\mathcal{W}}\boldsymbol{\mathcal{W}}^* = \boldsymbol{\mathcal{W}}^*\boldsymbol{\mathcal{W}} = \boldsymbol{I}_{N_{RX}} \quad (9)$$

The entries of $\boldsymbol{F}_{BB}$ are also normalised as in equation (10). This normalisation is made to ensure a total power constraint holds at the BS transmitter.

$$\|\boldsymbol{F}_{RF}\boldsymbol{F}_{BB}\|_F^2 = N_S \quad (10)$$

The $N_{TX} \times 1$ radiated signal vector $\boldsymbol{x}$ is given by equation (11),

$$\boldsymbol{x} = \boldsymbol{F}_{RF}\boldsymbol{F}_{BB}\boldsymbol{s}. \quad (11)$$

The $N_{RX} \times 1$ received symbol vector $\boldsymbol{y}_u = [y_{u,1}, y_{u,2}, ...y_{u,N_{RX}}]^T$ observed at the $u^{th}$ MS is given by equation (12),

$$\boldsymbol{y}_u = \boldsymbol{H}_u\boldsymbol{x}+\boldsymbol{n}_u = \boldsymbol{H}_u\sum_{k_1=1}^{U}\sum_{k_2=1}^{M}\boldsymbol{F}_{RF}\boldsymbol{f}_{k_1,k_2}^{BB}s_{k_1,k_2}+\boldsymbol{n}_u, \quad (12)$$

where $\boldsymbol{H}_u$ is the spatial channel matrix of dimensions $N_{RX} \times N_{TX}$ and $\boldsymbol{n}_u \sim \mathcal{N}(\boldsymbol{0}, \sigma_n^2\boldsymbol{I}_{N_{RX}})$, $\sigma_n^2$ is the noise variance. $\boldsymbol{H}_u$ is assumed to be deterministic or constant for the duration of beam training. The received symbol vector $\boldsymbol{y}_u$ is processed through $N_{RX} \times M$ RF combiner $\boldsymbol{W}_u^{RF} = [\boldsymbol{w}_{u,1}, \boldsymbol{w}_{u,2}, ..., \boldsymbol{w}_{u,M}]$ and then by $M \times M$ baseband digital combiner $\boldsymbol{W}_u^{BB} = [\boldsymbol{w}_{u,1}^{BB}, \boldsymbol{w}_{u,2}^{BB}, ..., \boldsymbol{w}_{u,M}^{BB}]$. The entries of $\boldsymbol{W}_u^{RF}$ are normalised such that $|[\boldsymbol{W}_u^{RF}]_{m,n}|^2 = N_{RX}^{-1}$. This letter focuses on noise performance analysis of RF beamforming, so we may assume that $\boldsymbol{F}_{BB} = \boldsymbol{I}_{N_s}$ and $\boldsymbol{W}_u^{BB} = \boldsymbol{I}_M$. Then, the received signal $r_{u,m}$ for $m^{th}$ stream of the $u^{th}$ MS is given by equation (13),

$$r_{u,m} = \boldsymbol{w}_{u,m}^* \ \boldsymbol{y}_u = \boldsymbol{w}_{u,m}^*\boldsymbol{H}_u\boldsymbol{x} + \boldsymbol{w}_{u,m}^*\boldsymbol{n}_u. \quad (13)$$

# III. Problem Statement

For the $m^{th}$ stream of the $u^{th}$ user, in the first stage of the HBF algorithm [11], RF beamforming vectors $\boldsymbol{f}_{u,m}^{RF}$ and $\boldsymbol{w}_{u,m}$ are chosen from their respective orthogonal beamforming codebooks $\boldsymbol{\mathcal{F}}$ and $\boldsymbol{\mathcal{W}}$ such that $|k_{u,m}|$ is maximised, where $k_{u,m}$ is given by equation (14),

$$k_{u,m} = \boldsymbol{w}_{u,m}^*\boldsymbol{H}_u\boldsymbol{f}_{u,m}^{RF}. \quad (14)$$

It occurs when $\boldsymbol{f}_{u,m}^{RF}$ and $\boldsymbol{w}_{u,m}$ point close to the directions of angle of departure (AoD) $\phi_{u,m}$ and angle of arrival (AoA) $\varphi_{u,m}$ of the available $(u,m)^{th}$ MPC. $|k_{u,m}|^2$ is the combined transmit and receive beamforming gain with the maximum value of $N_{RX} \times N_{TX}$ when the transmit and receive beamforming MRAs match exactly with $\phi_{u,m}$ and $\varphi_{u,m}$. If there are angular offsets, scalloping loss [18] will be observed. For simplicity and without the loss of generality, we assume the former condition such that the combined beamforming gain (BFG) is equal to $N_{RX} \times N_{TX}$. For orthogonal RF beamforming codebooks $\boldsymbol{\mathcal{F}}$ and $\boldsymbol{\mathcal{W}}$, $|k_{u,m}|$ should be zero or very small for all other combinations of beamforming vectors for which the MPC does not exist. Due to the presence of receiver AWGN noise during the beam training phase, equation (14) becomes equation (15),

$$z_{u,m} = k_{u,m} + \boldsymbol{w}_{u,m}^*\boldsymbol{n}_u = \boldsymbol{w}_{u,m}^*\boldsymbol{H}_u\boldsymbol{f}_{u,m}^{RF} + \boldsymbol{w}_{u,m}^*\boldsymbol{n}_u. \quad (15)$$

Thus, the noise can cause $|z_{u,m}|$ to be maximum for such combination of $\boldsymbol{f}_{u,m}^{RF}$ and $\boldsymbol{w}_{u,m}$ for which MPC does not exist. This case of false RF beam training leads to loss of communication due to reduced channel gain $|k_{u,m}|^2$ as seen at the baseband.

For exhaustive beam search, all $N_{TX} \times N_{RX}$ possible tx-rx beam combinations are tested for maximum receiver output. For multi-level beam search, progressively narrower beams are used in multiple levels of beam training to search the best tx-rx beam combination only for the angular sector identified by the preceding level beam search [13], [14]. Multi-level beam search is also known as hierarchical or tree-based beam training.

It is imperative to quantify the communication loss with respect to different values of SNR and MIMO dimensions due to false beam selections for both exhaustive and hierarchical RF beam training algorithms. Characterisation of such communication loss with the MIMO dimensions and SNR will help to determine the optimum MIMO configuration and transmitter power level for various mmwave communication scenarios.

## IV. Analytical Derivations

We derive closed-form expressions for the Exhaustive beam search first in subsection IV.I and then for the Hierarchical beam search in subsection IV.II.

### IV.1. Exhaustive Beam Search

Since the noise is i.i.d., the probability of the correct beam training by selecting the tx-rx beam combination, from $N_{TX} \times N_{RX}$ possible beam combinations, corresponding to the MPC of the channel such that $|z_{u,m}|$ is maximised can be given by the product of probabilities for $(N_{TX} \times N_{RX}) - 1$ pairwise correct beam selections. The problem thus reduces to a two-dimensional scenario.

We assume signal stream The bivariate probability density function for two independent Gaussian random variables $X_1 \sim \mathcal{N}(0, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(0, \sigma_2^2)$ is given by equation (16),

$$f_{X_1,X_2}(x_1, x_2) = f_{X_1}(x_1) f_{X_2}(x_1) = \frac{1}{2\pi\sigma_1\sigma_2} e^{-x_1^2/2\sigma_1^2}\; e^{-x_2^2/2\sigma_2^2}. \quad (16)$$

We take only one random sample $\mathrm{x}_1$ and $\mathrm{x}_2$ each from $X_1$ and $X_2$, respectively. The probability of $|\mathrm{x}_1| > |\mathrm{x}_2|$ is given by equation (17),

$$\begin{aligned} P(|\mathrm{x}_1| > |\mathrm{x}_2|) &= 2\int_0^\infty \int_{-\mathrm{x}_1}^{\mathrm{x}_1} f_{X_1,X_2}(x_1, x_2)\, dx_2\, dx_1 \\ &= 2\int_0^\infty \int_{-\mathrm{x}_1}^{\mathrm{x}_1} \frac{1}{2\pi\sigma_1\sigma_2} e^{\frac{-1}{2}\left(\frac{x_1^2}{\sigma_1^2}+\frac{x_2^2}{\sigma_2^2}\right)} dx_2\, dx_1 \\ &= 2\int_0^\infty \frac{1}{\sqrt{2\pi}\sigma_1} e^{-x_1^2/2\sigma_1^2} \mathcal{A}\, dx_1, \quad (17) \end{aligned}$$

where $\mathcal{A}$ can be evaluated by the error function and is given by equation (18),

$$\mathcal{A} = \mathrm{erf}\left(\frac{\mathrm{x}_1}{\sqrt{2}\sigma_2}\right). \quad (18)$$

Substituting equation (18) in equation (17) gives equation (19),

$$P(|\mathrm{x}_1| > |\mathrm{x}_2|) = 2\frac{1}{\sqrt{2\pi}\sigma_1} \int_0^\infty e^{-x_1^2/2\sigma_1^2}\, \mathrm{erf}\left(\frac{x_1}{\sqrt{2}\sigma_2}\right) dx_1. \quad (19)$$

Using identity (2) of section 4.3 of [17], we get equation (20),

$$\int_0^\infty e^{-x_1^2/2\sigma_1^2} \mathrm{erf}\left(\frac{x_1}{\sqrt{2}\sigma_2}\right) dx_1 = \sigma_1 \left(\frac{\pi}{2}\right)^{0.5} \left(1 - \frac{2}{\pi}\tan^{-1}\left(\frac{\sigma_2}{\sigma_1}\right)\right). \quad (20)$$

Equation (20) in equation (19) gives equation (21),

$$P(|\mathrm{x}_1| > |\mathrm{x}_2|) = 1 - \frac{2}{\pi}\tan^{-1}\left(\frac{\sigma_2}{\sigma_1}\right). \quad (21)$$

Let $X_1$ represent the case for the correct beam training so that $\sigma_1^2$ corresponds to the transmitted signal variance plus the noise variance plus the total beamforming power gain of $\mathrm{BFG} = N_{RX} \times N_{TX}$. Let $X_2$ represent the case for false beam training such that $\sigma_2^2$ corresponds only to the noise variance since no MPC exists for this beam combination and the correlation or power leakage from other beam-combinations is ideally zero due to the orthogonality of the beamforming vectors or very less in cases of angular offsets from the orthogonal MRAs. Then by equation (21), the probability of correct beam training $P_{cbt2}$ is given by equation (22),

$$\begin{aligned} P_{cbt2} &= 1 - \frac{2}{\pi}\tan^{-1}\left(\frac{\sigma_2}{\sigma_1}\right) \\ &= 1 - \frac{2}{\pi}\tan^{-1}\left(\frac{\sigma_n}{\sqrt{\sigma_s^2\ \mathrm{BFG} + \sigma_n^2}}\right) \\ &= 1 - \frac{2}{\pi}\tan^{-1}\left(\frac{1}{\sqrt{\mathrm{SNR} \times \mathrm{BFG} + 1}}\right), \quad (22) \end{aligned}$$

where $\mathrm{SNR} = \sigma_s^2/\sigma_n^2$.

$P_{cbt2}$ of equation (22) gives the probability of correct beam training for only two beam combinations. We noted earlier that for $N_{TX} \times N_{RX}$ beam combinations, the probability of correct beam training is given by the product of $N_{TX} \times N_{RX} - 1$ pairwise correct beam selections due to their independence. So, the probability of overall correct beam training $P_{cbt}$ is given by equation (23),

$$P_{cbt} = \left[1 - \frac{2}{\pi}\tan^{-1}\left(\frac{1}{\sqrt{1 + \mathrm{SNR} \times \mathrm{BFG}}}\right)\right]^N, \quad (23)$$

where $N = N_{TX} \times N_{RX} - 1$.

The mean communication rate $\overline{R}$ (bps/Hz) for noisy beam training is given by equation (24) ,

$$\overline{R} = P_{cbt}\ R, \quad (24)$$

where $R = \log_2(1+\mathrm{SNR}\times\mathrm{BFG})$ is the communication rate with ideal beam training i.e. without any false beam selections due to receiver noise.

The optimum SNR and BFG which in turn depends on the MIMO dimensions can be found by differentiating the $RL = R - \overline{R}$ with respect to SNR and BFG and equating to zero, respectively. For the minimum $RL$, the second derivative of $RL$ with respect to SNR and BFG must be positive.

### IV.2. Multi-level Beam Search

For multi-level beam search, let $k$ be the number of beam-search levels both at the BS and the MS for simplicity. For given $N_{TX}$ and $N_{RX}$, $k$ should be chosen such that it is an integer as it is the number of levels for the hierarchical beam search. For the first level, all possible combinations of $b_t$ transmit beams and $b_r$ receive beams will be tested for maximum received signal strength i.e. $k_1 = b_t \times b_r - 1$ beam combinations will be tested where $b_t$ and $b_r$ are $k^{th}$ roots of $N_{TX}$ and $N_{RX}$, respectively. If BS and MS employ different numbers of beam-search levels, say $k_t$ and $k_r$, respectively, then $b_t$ and $b_r$ will be $k_t^{th}$ and $k_r^{th}$ roots of $N_{TX}$ and $N_{RX}$, respectively. Since BS typically can have much larger number of antenna elements as compared to MS, $k_t > k_r$. In those cases, the receive beam selected for $k_r^{th}$ level will be used for higher levels of transmit beam search. For notational simplicity and with only trivial changes otherwise, we go ahead with $k_t = k_r = k$.

Since unitary beamforming codebooks are used, $k_2 = (b_t + 1) \times (b_r + 1) - 1 = k_1 + b_t + b_r$ beam combinations will be tested for second and further levels to include beams at both edges of the respective sector. The receiver noise power remains the same for all the levels. The signal and noise statistics are the same as for the exhaustive beam search. The combined beamforming gain for different levels will be $\text{BFG}/(b_t b_r)^{k-j}$ where $j$ is the beam search level number. Thus, the probability for correct beam training for tree-based beam search is given by equation (25),

$$P_h = \left[1 - \frac{2}{\pi}\tan^{-1}\left(\frac{1}{\sqrt{1 + \text{SNR} \times \text{BFG}/(b_t b_r)^{k-1}}}\right)\right]^{k_1} \times \prod_{j=2}^{j=k}\left[1 - \frac{2}{\pi}\tan^{-1}\left(\frac{1}{\sqrt{1 + \text{SNR} \times \text{BFG}/(b_t b_r)^{k-j}}}\right)\right]^{k_2}. \tag{25}$$

Equation (24) remains valid with $P_{cbt} = P_h$. If the MPC for the second stream is not in the same final level sector as the MPC for the first stream, then the probability of correct beam training for the second stream is given by $P_h$ of equation (25). If the two MPCs are in the same final level sector, then the probability of correct beam allocation for the second stream is given by equation (26),

$$P_{h2} = P_h \div \left[1 - \frac{2}{\pi}\tan^{-1}\left(\frac{1}{\sqrt{1 + \text{SNR} \times \text{BFG}}}\right)\right], \tag{26}$$

because one less beam combination needs to be tested in the final level with full beamforming gain since it has already been allocated to the first MPC.

For $N_{TX} = 64$, $N_{RX} = 8$ and $k = 3$, we get $N = 511$ and $K = k_1 + k_2 \times (k - 1) = 63$ for exhaustive and multi-level beam searches for the first stream, respectively. $N$ and $k_1, k_2$ are the exponents in equations (23) and (25), respectively. Since $N \gg K$, we see that noise gets much less chances of causing false beam selections for multi-level beam search as compared to exhaustive beam search. Hence, much higher mean communication rates are achievable with the hierarchical RF beam training than with exhaustive beam search.

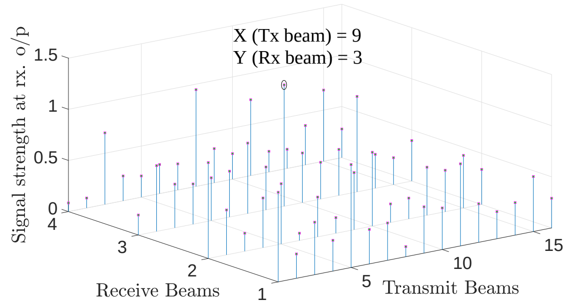


Figure 2. Noisy exhaustive beam search; $N_{RX} = 4$ $N_{TX} = 16$

## V. Simulation Results

Using equation (15), exhaustive beam search in presence of i.i.d. noise was implemented in MATLAB which is illustrated in Figure 2. In absence of noise i.e. with ideal beam training all beam combinations, except $9^{th}$ tx beam and $3^{rd}$ rx beam highlighted by a marker in the figure, should have produced zero output at the receiver due to orthogonality of the beams. However, a finite receiver output power is sensed due to noise for all the beam combinations as seen in the figure. This can lead to false beam selection which doesn't correspond to an actual MPC manifesting as communication loss.

The closed-form expressions derived were simulated in Matlab to obtain performance curves. Figure 3 shows the plots of the probabilities of correct beam selections obtained from equations (23) and (25) for exhaustive and multi-level beam selection algorithms, respectively. It is seen that hierarchical beam selection yields about 2.25 and 13 dB SNR gain for correct beam selection with 0.5 probability over exhaustive beam search for the two different MIMO dimensions. Such gain increases exponentially with MIMO dimensions and the number of search levels for tree-based i.e. hierarchical beam search.

Figure 4 presents the spectral efficiency curves for ideal and noisy RF beam training obtained from equation (24) for exhaustive and tree-based beam searches. It can be seen in Figure 4 that the spectral efficiency of noisy exhaustive beam training is almost equal to that of the ideal i.e. noise-less beam training below and above SNR thresholds A and B, respectively. It is very clearly seen that the communication loss between A and B is due to false beam selections as the probability for correct beam selection is very much less than one in the corresponding SNR region in Figure 3. The communication rate curve for noisy multi-level beam search approaches that of ideal beam search at point

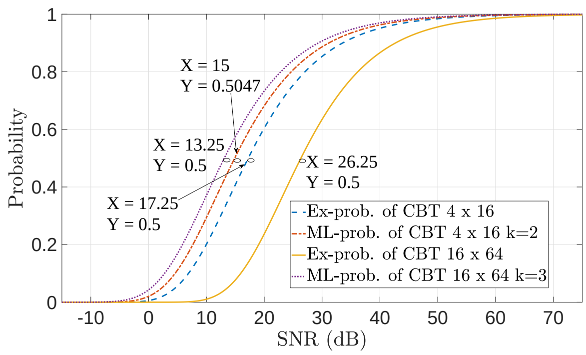


Figure 3. Probability curves for correct beam selections for Exhaustive and Multi-level beam searches

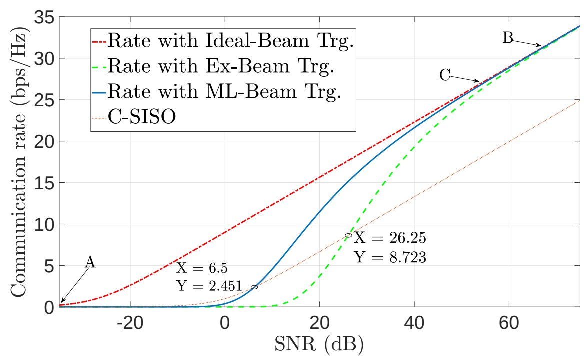


Figure 4. Communication rate curves for Exhaustive and Hierarchical beam searches; $N_{TX} = 64 \quad N_{RX} = 8 \quad k = 3$

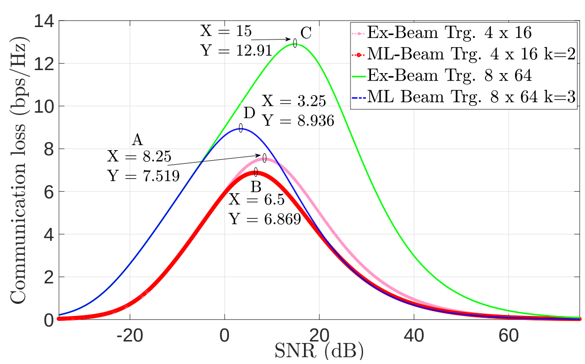


Figure 5. Performance comparison for Exhaustive and Hierarchical beam searches

C at a lower SNR than for point B. Also between the points A and C, the rate curve for multi-level beam search is much closer to that for ideal beam training than the rate curve obtained from exhaustive beam search.

In Figure 5, the communication losses ($RL = R - \overline{R}$) with reference to the ideal beam search for both beam training algorithms have been explicitly plotted for the two MIMO dimensions. Peak loss points are marked as A, B, C and D for respective plots. The area under the plots give the cumulative loss across all SNRs which is a reasonable indicator of the overall performance. For both the beam training algorithms, both the peak loss and the cumulative loss across all SNRs increase with MIMO dimensions as the number of beam combinations to be tested increases and the noise gets more opportunities to cause false beam selections. For exhaustive beam search, the peak loss has increased from point A to point C by about 5.4 bps/Hz with doubling of $N_{RX}$ and quadrupling of $N_{TX}$. For hierarchical beam search, the increase in peak loss from point B to point D is only 2.1 bps/Hz which is much smaller than that for exhaustive beam search. Overall saving in cumulative loss for tree-based beam search increases exponentially with increase in MIMO dimensions and the number of training levels. Thus, as we go towards higher MIMO dimensions the multi-level beam search becomes more effective in reducing the communication loss due to false beam selections owing to receiver noise.

As seen in Figure 5, peak loss point C for exhaustive beam search occurs at a higher SNR of about 7 dB than that of point A since communication rate with ideal beam training increases faster with increase in MIMO dimensions than that with exhaustive beam training. This happens despite the increase in BFG because with increased $N_{RX}$ and $N_{TX}$ noise gets more chances of causing false beam selections. This phenomenon is checked by multi-level beam training by restricting the number of beam combinations to be tested for correct beam allocation. As a result, point D is not only 4 bps/Hz lower than point C, but also occurs at about 12 dB smaller SNR.

Finally, the communication loss decreases with decrease in SNR for extremely low SNRs despite close-to-zero probability of correct beam training because the basic spectral efficiency with ideal beam training is itself extremely low due to Shannon's capacity equation and thus the communication loss also is negligibly low even with very high i.e. almost one probability of false beam selection.

## VI. Conclusion

Closed-form expressions have been derived to quantify the communication loss for noisy exhaustive and multi-level RF beam searches for mmwave massive MIMO systems using orthogonal beamforming codebooks. The performance of noisy beam training is lower than that of ideal beam training between the threshold SNR values which are closer for tree-based beam search than for the exhaustive beam search. With MIMO dimensions, cumulative and peak losses increase much faster for exhaustive beam search than for hierarchical beam search. Multi-level beam search algorithm helps mitigate communication loss as the chances for the receiver noise to cause false beam selection are very much reduced. With increasing MIMO dimensions the performance of hierarchical beam training gets exponentially closer to the noise-free beam search. For multi-level beam training, the SNR threshold values decrease with increase in MIMO dimensions and the levels of beam search. Subsequently trained streams have lower probability of false beam allocation and hence experience smaller losses. These results are planned to be validated by Monte-Carlo simulations close to real-life scenarios.

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## References

[1] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, Cambridge University Press, 2005.

[2] T. S. Rappaport, R.W. Heath Jr., R.C. Daniels, and J.N. Murdoch, *Millimeter Wave Wireless Communications*, Prentice Hall, 2014.

[3] V. Wong, R. Schober, D. Ng, and L. Wang, *Key Technologies for 5G Wireless Systems*, Cambridge University Press, 2017.

[4] T.S.Rappaport, et. al, Millimeter Wave Mobile Communications for 5G Cellular: It Will Work!, *IEEE Access Journal*, vol. 1, pp 335-349, May 2013.

[5] IEEE, "IEEE Standard - Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications Amendment 3: Enhancements for Very High Throughput in the 60 GHz Band (adoption of IEEE Std 802.11ad-2012)," in ISO/IEC/IEEE 8802-11:2012/Amd.3:2014(E), 14 March 2014.

[6] IEEE, "IEEE Standard - Part 15.3 : Wireless Medium Access Control (MAC) and Physical Layer (PHY) Specifications for High Rate Wireless Personal Area Networks (WPANs) Amendment 2: Millimeter-wave-based Alternative Physical Layer Extension," in IEEE Std 802.15.3c-2009 (Amendment to IEEE Std 802.15.3-2003), 12 2009.

[7] Silicon Image, "WirelessHD Specification Version 1.1," May 2010. [Online]. Available: http://www.wirelesshd.org/pdfs/WirelessHD-Specification-Overview-v1.1May2010.pdf [Accessed: 18-Jul-2018].

[8] A. M. Sayeed, "Deconstructing multiantenna fading channels," *IEEE Transactions on Signal Processing*, vol. 50, no. 10, pp. 2563-2579, Oct 2002.

[9] A. Sayeed and J. Brady, "Beamspace MIMO for high-dimensional multiuser communication at millimeter-wave frequencies," *IEEE Global Communications Conference (GLOBECOM)*, Atlanta, GA, pages 3679-3684, December 2013.

[10] G. H. Song, J. Brady, and A. Sayeed, "Beamspace MIMO transceivers for low-complexity and near-optimal communication at mm-wave frequencies," *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Vancouver, BC, pages 4394-4398, May, 2013.

[11] A. Alkhateeb, G. Leus, and R. W. Heath, "Limited feedback hybrid precoding for multi-user millimeter wave systems," in *IEEE Transactions on Wireless Communications*, vol. 14, no. 11, pp. 6481-6494, Nov. 2015.

[12] G. P. Fettweis and E. Zimmermann, "ICT Energy Consumption-Trends and Challenges," *The 11th International Symposium on Wireless Personal Multimedia Communications (WPMC 2008)*, Lapland, Finland, pages 1-4, September, 2008.

[13] S. Hur, T. Kim, D. J. Love, J. V. Krogmeier, T. A. Thomas, and A. Ghosh, "Multilevel millimeter wave beamforming for wireless backhaul," *IEEE GLOBECOM Workshops (GC Wkshps)*, Houston, TX, pages 253-257, December, 2011.

[14] S. Hur, T. Kim, D. J. Love, J. V. Krogmeier, T. A. Thomas, and A. Ghosh, "Millimeter Wave Beamforming for Wireless Backhaul and Access in Small Cell Networks," in *IEEE Transactions on Communications*, vol. 61, no. 10, pp. 4391-4403, October 2013.

[15] A. Alkhateeb and R. W. Heath, 'Frequency Selective Hybrid Precoding for Limited Feedback Millimeter Wave Systems," *IEEE Transactions on Communications*, vol. 64, no. 5, pp. 1801-1818, May 2016.

[16] H. L. V. Trees, *Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory*, New York: Wiley-Interscience, 2002.

[17] Edward W. Ng and Murray Geller, " Table of Integrals of the Error Functions," *JOURNAL OF RESEARCH of the Natianal Bureau of Standards - B. Mathematical Sciences*, vol. 73B, No. 1, January-March 1969.

[18] K. K. Tiwari, "Beamforming Techniques for Millimetre Wave Communications," *section 3.9, M. Sc. thesis, matriculation number s1113092*, The University of Edinburgh, Edinburgh, U.K., 2017.