

An Efficient and Low Complexity Greedy Power Allocation Algorithm for URLLC Links

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Abstract

In this paper, an efficient and low-complexity power allocation algorithm for a URLLC-link is proposed and analyzed based on a previously published formula for calculating the effective channel capacity. The underlying optimization problem differs from standard communication links because the well-known Shannon formula cannot be applied, and two additional rate penalties must be considered. The first penalty results from the short block length of the transmitted URLLC data packets, and the second penalty comes from the additional delay caused by the queueing process at the link layer. Two mathematical methods are applied to deal with these penalties: the Finite Block Length Information Theory (FBL-IT) and the Effective Capacity (EC) calculation. We explain this formula for calculating the effective channel rate, including the two mentioned constraints for URLLC communication. Furthermore, to approach a solution for the optimization problem, an efficient greedy algorithm for allocating the total transmit power over the available channel uses of an OFDM-based URLLC system is proposed and studied. Since the URLLC service will be shared with other communication services on the same link, the greedy algorithm will be used to allocate the total allowed power of the link to the different individual communication channels in such a way that their communication requirements can be fulfilled. For this purpose, the link is divided into groups of channels with the same properties, and the greedy algorithm is applied to each group individually. Simulation results show the efficiency and reliability of the proposed algorithm for both eMBB and URLLC links.

1 Introduction

The current Fifth-Generation (5G) wireless communication system is targeting to support three wireless use cases with different Quality of Service (QoS) requirements and traffic characteristics. The enhanced Mobile Broadband (eMBB) use case supports wireless services with a very high data rate, the massive Machine Type Communications (mMTC) use case supports wireless services for Machine-to-Machine (M2M) communication, and the Ultra Reliable Low latency Communications (URLLC) use case supports real-time communication for low latency applications such as industry automation and tactile internet. Wireless real-time communication for cellular systems like 4G or 5G shall be performed by URLLC. However, the required round-trip delay of < 1 ms cannot be achieved until now. The ongoing DFG project 5G-REMOTE investigates the underlying problem and suggests methods to overcome it.

The authors in [1] introduce a new mathematical tool called Effective Capacity (EC) as a link-layer model to address the challenge of providing QoS in delay-constrained wireless networks, which is critical for supporting diverse traffic types and meeting their QoS requirements. Traditional physical-layer channel models are insufficient as they do not explicitly account for QoS metrics like data rate, delay, and delay-violation probability. EC models the wireless link in terms of two functions: the probability of a nonempty buffer and the QoS exponent of a connection, offering a simple and efficient method for estimating these functions. This approach facilitates translating physical

channel characteristics into QoS guarantees, simplifying implementation and enhancing the accuracy and efficiency of access control and resource allocation processes. The effective capacity maps the wireless physical channel rate, derived from the Shannon formula, to effective wireless channel rate. The effective rate is not only a function of the received Signal to Noise (SNR), but it will be a function of the QoS exponent that reflects the effects of queueing delay and queueing delay violation probability. Depending on this seminal work of [1], many research work was done on deriving the effective capacity for various wireless communication systems. The authors in [2] comprehensively surveyed all the research work done on this new mathematical tool. They discussed the necessity of the EC approach for QoS in next-generation wireless networks, especially for low-latency wireless applications. The survey categorizes existing works on EC, detailing its use in cognitive radio networks, cellular networks, relay networks, ad-hoc networks, and mesh networks. The survey explored case studies, analyzed the EC in the context of different fading models, and looked at delay-sensitive applications. It outlined future research directions, focusing on maximizing EC and proposing efficient resource allocation and transmission designs. Polyanskiy et al. in [3] presented another new mathematical tool to study and derive the effective rate of the short packet transmission that couldn't be studied by the well-known Shannon formula. The authors presented an in-depth analysis of the maximum channel coding rates achievable with a given blocklength and error probability, providing new achievability and converse bounds. They

introduced tighter approximations for the maximal achievable rate over short blocklengths, significantly contributing to understanding the finite blocklength regime in information theory. They addressed the practical challenge of achieving reliable communication at finite blocklengths, crucial for modern communication systems demanding low latency and high reliability. Polyanskiy et al. in [4] reviewed recent advances in transmitting short packets, which are essential for future wireless systems, including 5G and beyond. They addressed the challenges of supporting novel traffic types requiring short packets, such as mMTC and URLLC. They also discussed the inadequacy of current wireless systems for short-packet transmissions and presented theoretical foundations and examples to illustrate the optimization of control information transmission in such scenarios. They suggested new principles for designing wireless protocols to efficiently support short packets, highlighting the impact of finite blocklength regimes on system design.

In our previous work, the two mathematical tools were used to derive the effective transmission rate for URLLC wireless communication systems [5]. In this paper, the effective rate will be used to formulate an optimization problem of maximizing the effective capacity of single user OFDM-based URLLC system model under the required delay and reliability constraints. An efficient and low complexity greedy algorithm is proposed to solve the optimization problem.

2 System Model and Problem Formulation

2.1 System Model

We focus on the discrete-time Orthogonal Frequency Division Multiplexing (OFDM) system over a point-to-point (single user case) wireless link between the transmitter and the receiver. Let us denote the system's total spectral bandwidth by B , the mean transmit power by P , and the power density of the complex additive white Gaussian noise (AWGN) by $N_0/2$. First, the upper-protocol-layer packets are divided into frames at the data link layer, which forms the "data source". We assume that the frames have the same time duration, which is denoted by T_f . The frames are stored at the transmit buffer and then split into bit-streams at the physical layer. Based on the QoS constraint and the channel-state information (CSI) fed back from the receiver, the adaptive modulation and power control are employed, respectively, at the transmitter. The reverse operations are executed at the receiver side. Finally, the frames are recovered at the "data sink" for further processing. The CSI is assumed to be perfectly estimated at the receiver and reliably fed back to the transmitter without delay.

2.2 Problem Formulation and the proposed greedy algorithms

We assume that there are two kinds of traffic. The traditional traffic of eMBB with long packets and loose delay QoS constraints. The second traffic comes from sensors and machines with short packets and stringent delay QoS constraints. There are two problems that can be formulated and solved as follows:

2.2.1 Rate Maximization Problem for eMBB

The well-known Shannon formula will be used to formulate the constrained Rate Maximization (RM) problem under the requirements of eMBB. The RM transmission optimization problem of the OFDM wireless communication system is considered, where the multi-path channel characterized by a Finite Impulse Response (FIR) vector $\mathbf{h} = [h_0 \ h_1 \ \cdots \ h_{L_p-1}]$ of order L_p is converted to an N -sub-carriers system with different gains $\mathbf{g}_n = |\mathbf{H}_n|$, $n = 1, 2, \dots, N$. The n^{th} sub-carrier experiencing the gain \mathbf{g}_n will be used to transmit R_n bits per subcarrier. The RM optimization problem of the single-user OFDM wireless communication system can be formulated as:

Objective

$$\max_{p_n} \sum_{n=1}^N R_n \quad (1.a)$$

Constraints

$$\begin{aligned} \text{C1: } & \sum_{n=1}^N p_n \leq P_t, \\ \text{C2: } & p_n \geq 0, \quad \forall n : 1 \leq n \leq N \\ \text{C3: } & \mathcal{P}_{e,n} \leq \mathcal{P}_{e,\text{target}}, \quad \forall n : 1 \leq n \leq N \\ \text{C4: } & R_n \leq R_{\max}, \quad \forall n : 1 \leq n \leq N \end{aligned} \quad (1.b)$$

where R_n and p_n are the rate and power allocated to the n^{th} sub-carrier to achieve a BER of $\mathcal{P}_{e,n}$, P_t is the total transmit power, $\mathcal{P}_{e,\text{target}}$ is the system target BER, and R_{\max} is the maximum number of permissible allocated bits per sub-carrier.

The Channel to Noise Ratio (CNR) of the n^{th} sub-carrier can be defined as follows:

$$\text{CNR}_n = g_n^2 / \sigma_z^2 \quad (2)$$

where σ_z^2 is the noise power at the receiver, and the SNR of the n^{th} sub-carrier is

$$\gamma_n = p_n \times \text{CNR}_n \quad (3)$$

Symbols of $R_l = \log_2 M_l$ bits can be allocated to the n^{th} sub-carrier with the minimum required SNR γ_l^{QAM} of

$$\gamma_l^{\text{QAM}} = \begin{cases} \frac{[Q^{-1}(P_{e,\text{target}})]^2}{2} & \text{for BPSK} \\ \frac{M_l - 1}{3} \left[Q^{-1} \left(\frac{1 - \sqrt{1 - \log_2 M_l \cdot P_{e,\text{target}}}}{2(1 - 1/\sqrt{M_l})} \right) \right]^2 & \text{for M-QAM} \end{cases} \quad (4)$$

where M_l is the maximum permissible QAM constellation by the transmission system and Q^{-1} is the inverse of the well-known Q function that is the tail probability of the standard normal distribution.

An efficient greedy power allocation algorithm is proposed to find the optimal rate and power allocation for the maximum achievable transmission rate. The greedy algorithm is started by Uniform Power Allocation (UPA) to alleviate the computational complexity, as follows:

- 1) Calculate γ_l^{QAM} for all M_l , $1 \leq l \leq L$, with target BER $\mathcal{P}_{e,n} = \mathcal{P}_{e,\text{target}}$
- 2) Allocate the total power budget. P_t between all sub-carriers equally, as follows

$$\gamma_n = p_n \times \text{CNR}_n = \frac{P_t}{N} \times \frac{|H_n|^2}{\sigma_z^2} \quad (5)$$

- 3) Redistribute sub-carriers according to their SNRs γ_n into QAM groups G_l , $0 \leq l \leq L$ bounded by QAM levels γ_l^{QAM} and $\gamma_{l+1}^{\text{QAM}}$ with $\gamma_0^{\text{QAM}} = 0$ and $\gamma_{L+1}^{\text{QAM}} = +\infty$ i.e.

$$\gamma_l^{\text{QAM}} \leq \gamma_n < \gamma_{l+1}^{\text{QAM}} \quad (6)$$

- 4) Load sub-carriers within each G_l group with QAM constellation M_l such that the total number of allocated bits of this group is

$$R_l^{\text{UPA}} = \sum_{n \in G_l} R_n \quad (7)$$

with $R_0^{\text{UPA}} = 0$ and the total excess power for this group given as:

$$P_{l,\text{excess}}^{\text{UPA}} = \sum_{n \in G_l} \frac{(\gamma_n - \gamma_l^{\text{QAM}})}{\text{CNR}_n} \quad (28)$$

- 5) The total number of the system allocated bits and the power of the UPA algorithm are

$$R^{\text{UPA}} = \sum_{l=1}^L R_l^{\text{UPA}} \quad (9)$$

$$P_{\text{alloc}}^{\text{UPA}} = P_t - P_{\text{excess}}^{\text{UPA}} = P_t - \sum_{l=1}^L P_{l,\text{excess}}^{\text{UPA}} \quad (10)$$

Then, the proposed Greedy Power Allocation (GPA) Algorithm can be summarized in Table (1):

Table (1) The GPA algorithm	
Step	Operation
Input	R_n^{UPA} , $P_{\text{excess}}^{\text{UPA}}$, and γ_n , $1 \leq n \leq N$
Initialization	<ul style="list-style-type: none"> Set the excess power of the GPA algorithm, $p_{\text{excess}}^{\text{GPA}} = P_{\text{excess}}^{\text{UPA}}$ for each sub-carrier do the following: <ol style="list-style-type: none"> Set $R_n^{\text{GPA}} = R_n^{\text{UPA}}$ Initiate $l_n = l$ <p>Calculate the minimum required upgrade power: $p_n^{\text{up}} = \frac{(\gamma_{l_n+1}^{\text{QAM}} - \gamma_{l_n}^{\text{QAM}})}{\text{CNR}_{l_n}}$</p>
Iteration	<p>while $P_{\text{excess}}^{\text{GPA}} \geq \min(p_n^{\text{up}})$ and $\min(l_n) < L$</p> <p>$j = \arg \min(p_n^{\text{up}})$, $1 \leq n \leq N$</p> <p>$l_j = l_j + 1$, $P_{\text{excess}}^{\text{GPA}} = P_{\text{excess}}^{\text{GPA}} - p_j^{\text{up}}$</p> <p>If $l_j = 1$</p> $R_j^{\text{GPA}} = \log_2 M_1, p_j^{\text{up}} = \frac{(\gamma_2^{\text{QAM}} - \gamma_1^{\text{QAM}})}{\text{CNR}_j}$ <p>elseif $l_j < L$</p> $R_j^{\text{GPA}} = R_j^{\text{GPA}} + \log_2 \left(\frac{M_{l_j}}{M_{l_j-1}} \right), p_j^{\text{up}} = \frac{(\gamma_{l_j+1}^{\text{QAM}} - \gamma_{l_j}^{\text{QAM}})}{\text{CNR}_j}$ <p>else</p> $R_j^{\text{GPA}} = R_j^{\text{GPA}} + \log_2 \left(\frac{M_{l_j}}{M_{l_j-1}} \right), p_j^{\text{up}} = +\infty$ <p>end</p> <p>end</p> $R^{\text{GPA}} = \sum_{n=1}^N R_n^{\text{GPA}}$
Output	R^{GPA} , $P_{\text{excess}}^{\text{GPA}}$

2.2.2 Rate Maximization Problem for URLLC

The effective rate of the URLLC was studied and derived in our previous work[5]. In our paper, the achievable rate of short packets of URLLC will be used to find its effective capacity that reflects the required QoS of URLLC. There are two rate penalties resulting from the channel dispersion of the short packet transmission and the delay of the queueing process.

The achievable rate of the short packets of URLLC can be approximated as in [3]

$$R^*(n, \epsilon) = C - \sqrt{\frac{V}{n}} Q^{-1}(\epsilon) \quad (11)$$

Where n , ϵ , V are the packet length, probability of error, and channel dispersion, respectively.

The effective capacity of the achievable rate and that reflects the delay characteristics of the queueing process can be found as in

$$E_C(\theta) = -\frac{1}{\theta} \log(\mathbb{E}\{e^{-\theta R^*(n, \epsilon)}\}) \quad (12)$$

Where θ is the delay QoS exponent and is function of the delay (denominator) and delay violation probability (numerator) as in [6]

$$-\lim_{x \rightarrow \infty} \frac{\log(\Pr\{Q(\infty) > x\})}{x} = \theta \quad (13)$$

The optimization problem in eq. (1) will be modified such that the objective function will be replaced by the effective capacity in eq.(12), and the constraints functions will be modified to include the delay violation probability of eq.(13). Then, the problem will be solved with a modified

version of the GPA algorithm used in subsection 2.2.1 by the following steps:-

1. Finding the equivalent SNRs of each M constellation, as in eq.(4)
2. Using these SNRs to find their equivalent effective capacities, as in eq.(12)
3. Finding the new equivalent SNRs and using them in the GPA to find the optimal rate and power allocation for URLLC.

3 Simulation Results

The GPA of eMBB and the modified GPA of URLLC will be evaluated and compared by MATLAB simulations. The main simulation parameters are $BER = 10^{-9}$, subcarriers number is 64, square M-QAM constellations of $M = [4, 16, 64, 256]$, delay violation probability of 10^{-9} .

Fig.(1) shows a throughput comparison study between long packets' eMBB and short packets' URLLC transmissions. Ten thousand channel realizations are generated to calculate the mean throughput of the OFDM system under different power allocation algorithms, the UPA and the GPA, respectively.

The throughput, the total allocated bits per OFDM symbol, increases with the SNR. The GPA algorithm outperforms the UPA algorithm because it optimally allocates the bits and the power such that the good subcarriers are firstly allocated, then the bad subcarriers, and the worst subcarriers that suffer from deep fading will not be allocated any power or bits.

At high SNR, the throughput will be saturated at a fixed number, which is the multiplication of 64 subcarriers by the data bits in the highest M-QAM. In eMBB, the fixed throughput is 512 bits/symbol, i.e. 64×8 . In URLLC with $\theta = 2$, the fixed throughput is 256 bits/symbol, i.e. 64×4 . In URLLC with $\theta = 4$, the fixed throughput is 128 bits/symbol, i.e. 64×2 .

Fig.(1) also shows that the throughput performance of URLLC is less than that of eMBB. In URLLC, there are two rate penalties. One comes from the channel dispersion due to the short packet length of 64 QAM symbols, and the other comes from the delay characteristics of the queueing

process. The delay violation probability is assumed here to be equal to the $BER = 10^{-9}$, and the delay QoS exponent takes large values which are modified between two and four to express the stringent delay constraints of URLLC.

4 Conclusions and future work

An efficient and low-complexity GPA algorithm has been proposed to maximize the rate of eMBB and the effective capacity of URLLC. The complexity of the GPA algorithm has been alleviated by first using UPA. The GPA has improved the reliability of the transmission by avoiding allocating any power or bit to those subcarriers suffering deep fading. The modified GPA guarantees stringent delay constraints for URLLC with high-reliability transmission. The eMBB and URLLC have been individually studied and evaluated, but they will be jointly studied and evaluated in our future work.

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6 References

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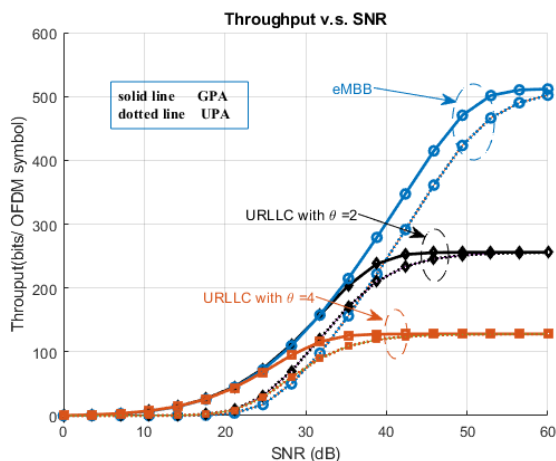


Figure 1 Throughput comparison study between eMBB and URLLC