

RESEARCH ARTICLE

Mid-Infrared Hyperbolic Metamaterial for the Silicon–Germanium Technology Platform

Jon Schlipf¹  | Enrico Talamas Simola²  | Fritz Berkmann³  | Damiano Marian⁴ | Inga A. Fischer⁵ | Francesco Bisio⁶ | Michele Ortolani³  | Leonetta Baldassare³ | Luciana di Gaspare² | Giovanni Capellini^{1,2}  | Monica De Seta² | Michele Virgilio⁴

¹IHP - Leibniz Institute for High Performance Microelectronics, Frankfurt (Oder), Germany | ²Dipartimento di Scienze, Università degli Studi Roma Tre, Roma, Italy | ³Dipartimento di Fisica, Sapienza Università di Roma, Roma, Italy | ⁴Dipartimento di Fisica “E Fermi”, Università di Pisa, Pisa, Italy | ⁵Chair of Experimental Physics and Functional Materials, BTU Cottbus-Senftenberg, Cottbus, Germany | ⁶CNR-SPIN, Genova, Italy

Correspondence: Enrico Talamas Simola (enrico.talamassimola@uniroma3.it)

Received: 7 May 2026 | **Revised:** 11 July 2026 | **Accepted:** 26 August 2026

Keywords: chemical vapor deposition | doping | germanium | materials science | metamaterial | mid infrared | optoelectronics | photonics | semiconductor

ABSTRACT

Hyperbolic metamaterials (HMMs) offer unprecedented control over light-matter interactions, yet integrating them into standard semiconductor platforms remains challenging. Here, we present the realization and optical characterization of a mid-infrared HMM based entirely on the silicon-germanium technology platform. The monolithic structure consists of alternating heavily doped (metal-like) and intrinsic (dielectric-like) germanium layers grown via CMOS-compatible ultra-high vacuum chemical vapor deposition (UHV-CVD). Using angle-resolved polarized reflectivity measurements, we experimentally demonstrate the emergence of a hyperbolic dispersion regime. The experimental optical response is accurately reproduced by a full multilayer model, which highlights the critical role of material losses when approaching the HMM spectral window. By simply adjusting the doping profile, this platform offers high design tunability and straightforward integration with mid-infrared photonic and plasmonic devices based on group-IV materials.

1 | Introduction

Metamaterials are artificial optical media, structured at the wavelength scale to engineer electromagnetic properties which do not typically occur in nature. “Left-handed” or counter-propagating materials belong to this broad class and were first theoretically predicted by Veselago [1], who realized that isotropic media with negative real parts of both dielectric and magnetic permittivities would feature peculiar optical phenomena such as wave propagation with phase velocity antiparallel to the Poynting vector, negative refraction, inverted Doppler and Čerenkov effects, and the possibility to fabricate a “perfect lens” [2, 3]. A key requirement to achieve negative permittivity and permeability is the coexistence of electric and magnetic

resonances in the same spectral region. This condition was experimentally achieved decades after the seminal paper of Veselago in the microwave domain, confirming many of his theoretical predictions [4].

Subsequently, it became clear that the double-resonance condition can be relaxed, relying instead on non-magnetic uniaxial anisotropic media, provided that the dielectric permittivity along the in-plane ($\epsilon_{||}$) and axial ($\epsilon_{\perp}$) directions have opposite signs [5]. In this case, the dispersion relation leads to hyperbolic isofrequency surfaces, hence the name hyperbolic metamaterials (HMM). The hyperbolic character grants a number of practical applications such as the propagation of waves with very large in-plane wave vectors that would be evanescent in conventional

All rights reserved, including rights for text and data mining and training of artificial intelligence technologies or similar technologies.

© 2026 Wiley-VCH GmbH

media, thereby enabling subdiffraction imaging or the broadband enhancement of the spontaneous emission via Purcell effect [6, 7].

Hyperbolic behavior can be achieved, among other possibilities [8], in periodic multilayered planar structures, consisting of alternating metallic and dielectric layers with thicknesses well below the wavelength scale. Using an effective medium approximation (EMA) to describe the anisotropic dielectric tensor of these uniaxial multilayers in terms of those of the constituents, one finds that the hyperbolic response emerges in the vicinity of the plasma frequency ω_p of the metallic material [9]. Early implementations of multilayer HMMs combined dielectric layers with noble metals featuring plasmonic resonances in the ultraviolet (UV) regime [10]. These initial successes motivated further research efforts to shift the HMM spectral window in the mid-infrared (MIR), due to its relevance for a number of practical applications. In the MIR, however, metals exhibit strongly negative permittivities, causing both $\epsilon_{\perp}$ and $\epsilon_{\parallel}$ to become negative, thereby precluding the emergence of hyperbolic behavior. Hence, metallic layers have been substituted by heavily-doped semiconductors which feature plasma frequencies in the MIR, leading to the demonstration of HMM around $\approx 10 \mu\text{m}$, first using InGaAs/AlInAs heterostructures [11], and subsequently leveraging single-material multilayers, realized by alternating doped and intrinsic InAs layers [12].

The hyperbolic behavior can be verified through a transmittance or a reflectance measurement. In the former case, it is inferred from geometric optics experiments, evidencing the opposite propagation direction with respect to the normal to the surface of TE and TM polarized beams passing through a slab of HMM [11–13]. In the latter case, one typically measures the dichroic reflectance ratio $R_{\text{TM}}/R_{\text{TE}}$ as a function of the beam's incidence angle, tracking the Brewster angle θ_B as a function of wavelength and identifying a discontinuity that signals the onset of the hyperbolic regime [13, 14].

Several authors [13] further exploited oblique-incidence reflectance spectra for TE and TM polarizations to fit the spectral dependence of the anisotropic dielectric tensor. For the fitting procedure to converge to a unique solution, it is necessary to neglect material losses by setting the imaginary part of the permittivity to zero. However, this approximation not only reduces accuracy but also becomes physically inconsistent in the vicinity of the plasma resonance. Furthermore, medium losses may greatly smooth the discontinuity of θ_B used to identify the frequency boundary of the hyperbolic regime (notice that in the presence of losses $R_{\text{TM}}/R_{\text{TE}}(\theta_B)$ is minimized without vanishing). Thus, a more rigorous consideration of the role of optical losses is required [9, 14, 15]. This would also be beneficial at the material design stage to optimize the HMM figures of merit as reported, for instance, in Ref. [15].

Large-scale deployment of HMM functionalities in the MIR ultimately requires compatibility with silicon photonics foundry technology, which so far has been hindered by the use of III–V compounds. To this end, Ge-rich SiGe heterostructures deposited on a Si substrate represent a promising material platform, given that they allow the realization of MIR integrated photonic circuits by leveraging fabrication techniques developed for silicon [16, 17]. Indeed, heavily doped Ge layers can support plasmonic

resonances in the MIR [18], thus, in principle, enabling single-material MIR HMMs based on alternating intrinsic and doped Ge layers. However, this platform also introduces significant material-related challenges. In particular, due to the relatively large effective mass of L-valley electrons in Ge ($m^* = 0.12$) compared to III–V compounds ($m^* < 0.1$), higher doping levels are required to achieve comparable plasma frequencies ω_p . This, in turn, leads to increased carrier scattering and larger damping constants, which broaden the plasmonic resonance and hinder the attainment of pronounced negative values of the anisotropic effective permittivity. As a consequence, optical losses emerge as a key limiting factor, making a rigorous, loss-inclusive description essential for the design of Ge-based MIR HMMs. These requirements pose additional challenges from a material-growth perspective, demanding micron-thick Ge/Ge⁺⁺ multilayers with carrier concentrations on the order of 10^{19} cm^{-3} , while maintaining precise control over dopant activation and spatial confinement. In practice, achieving such control remains difficult, as diffusion, clustering, and compensation effects tend to degrade the intended doping profiles, thereby creating further obstacles to the implementation of this platform.

In this work, we address these challenges by realizing a Ge-rich SiGe-based hyperbolic metamaterial through the fabrication of periodically alternating doped and undoped SiGe layers on germanium virtual substrates grown on silicon, targeting MIR hyperbolic behavior. We probe the hyperbolic character through grazing-incidence, polarization-resolved reflectance measurements in the mid-infrared, performed using Fourier-transform infrared (FTIR) spectroscopy with focused beams. To support the experimental findings and extract the effective anisotropic response, we rely on both analytical modeling based on the EMA with fully numerical simulations, explicitly accounting for the role of optical losses and reflection stemming from the buffer layer. Our results indicate the emergence of hyperbolic behavior in the wavelength range between approximately 12.5 and 15 μm , corresponding to a spectral bandwidth of about 20%.

2 | Ge/Ge⁺⁺ HMM Design and Growth

We start by numerically exploring the design parameter space, in order to identify a robust MIR hyperbolic behavior for wavelengths shorter than 15 μm . To this aim we first calculate the complex isotropic permittivity of Ge $\epsilon(\omega)$ using the Drude model

$$\epsilon(\omega) = \epsilon_{\infty} - \frac{\omega_p^2}{\omega^2 + i\omega\gamma}. \quad (1)$$

In the above equation $\epsilon_{\infty} = 16.7$ and $\omega_p^2 = \frac{N_D e^2}{\epsilon_0 m_{eff}}$ is the squared plasma frequency, controlled by the active donor concentration N_D and the L-point (001) transport mass m_{eff} , for which we use the value of $0.12m_0$ [19]. The damping factor γ is estimated from the electron mobility (μ), which in our system is limited by charged impurity (μ_i) and phonon scattering (μ_{ph}) according to Matthiessen's rule. We calculate μ_i as a function of N_D following Conwell and Weisskopf [20]. For μ_{ph} we use the value of 3600 cm^2/Vs , corresponding to Germanium layers with a background doping density of the order of 10^{15} cm^{-3} at room temperature [21, 22]. Varying N_D in the $(1\text{--}3) \times 10^{19} \text{ cm}^{-3}$ interval, we find that γ

spans the $(4\text{--}6)\times 10^{13}\text{ s}^{-1}$ range, in agreement with FTIR data from epitaxial Ge at similar doping densities reported in Ref. [18].

In the framework of the EMA, the anisotropic permittivity tensor of the multilayer stack is obtained in terms of the scalar permittivities of the doped (ϵ_m) and undoped (ϵ_d) layers, imposing the continuity of the parallel (vertical) component of the electric (displacement) field at the interface between these two regions. Following this procedure one finds

$$\epsilon_{||} = (1 - f)\epsilon_d + f\epsilon_m \quad (2)$$

$$\epsilon_{\perp} = \epsilon_m\epsilon_d / (f\epsilon_d + (1 - f)\epsilon_m) \quad (3)$$

where the filling factor f represents the relative thickness of the doped region with respect to the multilayer period.

The HMM design is typically optimized using as figure of merit $\text{FOM} = |\text{Re}(k_{\text{TM}})|/|\text{Im}(k_{\text{TM}})|$ i.e. the ratio between the real and imaginary parts of the TM polarized wave-vector component along the optical axis, which characterizes propagation losses in the hyperbolic regime [11]. Here, instead, since we probe the system through reflectivity measurements, we tune the design parameters to maximize the ‘depth’ D of the HMM spectral region, defined as the peak value of $\text{Re}(\epsilon_{||}(\omega)) - \text{Re}(\epsilon_{\perp}(\omega))$ within the frequency range where $\text{Re}(\epsilon_{||}) > 0$ and $\text{Re}(\epsilon_{\perp}) < 0$, setting $D = 0$ elsewhere. This reflectance-oriented metric is less sensitive than the conventional FOM to uncertain loss parameters and propagation geometry, an important consideration given the lack of previous SiGe HMM demonstrations and the scarcity of optical data for highly doped SiGe alloys. It also allows us to target a broad and well-defined hyperbolic region rather than designs close to the boundary where $\text{Re}(\epsilon_{\perp})$ only marginally becomes negative. In Figure 1a, we show the HMM depth as a function of the design parameters f and N_D ; a similar plot for the relative bandwidth is shown in the Supporting Information. D peaks at about 13 for a filling factor of 0.6 and a doping density of $\approx 1.5 \times 10^{18}\text{ cm}^{-3}$, corresponding to a wavelength of transition λ_0 between the dielectric and the hyperbolic regime at $\sim 40\text{ }\mu\text{m}$. The area on the right side of the green curve evidences the (f, N_D) subset where the wavelength onset of the HMM regime λ_0 occurs at wavelengths shorter than $15\text{ }\mu\text{m}$, corresponding to the low frequency cut-off of our experimental set-up. λ_0 as a function of (f, N_D) is reported in Figure 1b from which it is apparent that the $\lambda_0 < 15\text{ }\mu\text{m}$ MIR window requires doping densities at the 10^{19} cm^{-3} scale. In principle, shorter λ_0 could be achieved using larger doping densities but this comes with an increase of the scattering rate γ which smears the plasma resonance and induces less negative values of $\epsilon_{\perp}$, effectively lowering D .

Focusing on the $\lambda_0 < 15\text{ }\mu\text{m}$ region, Figure 1a suggests that D peaks at ≈ 7 , when the filling factor is 0.65, the active doping density $1.8 \times 10^{19}\text{ cm}^{-3}$, and $\lambda_0 = 12.1\text{ }\mu\text{m}$. For this choice of design parameters, the spectral shape for the real parts of $\epsilon_{||}$ and $\epsilon_{\perp}$ is shown in panel (c) of Figure 1 while their imaginary parts are reported in panel (d) of the same figure; the conventional FOM for various angles of incidence is included in the Supporting Information. From Figure 1c, we find that the region where $\epsilon_{||} > 0$ and $\epsilon_{\perp} < 0$, corresponding to the so-called type I hyperbolic dispersion, is spanning approximately $12\text{--}15\text{ }\mu\text{m}$ with a bandwidth

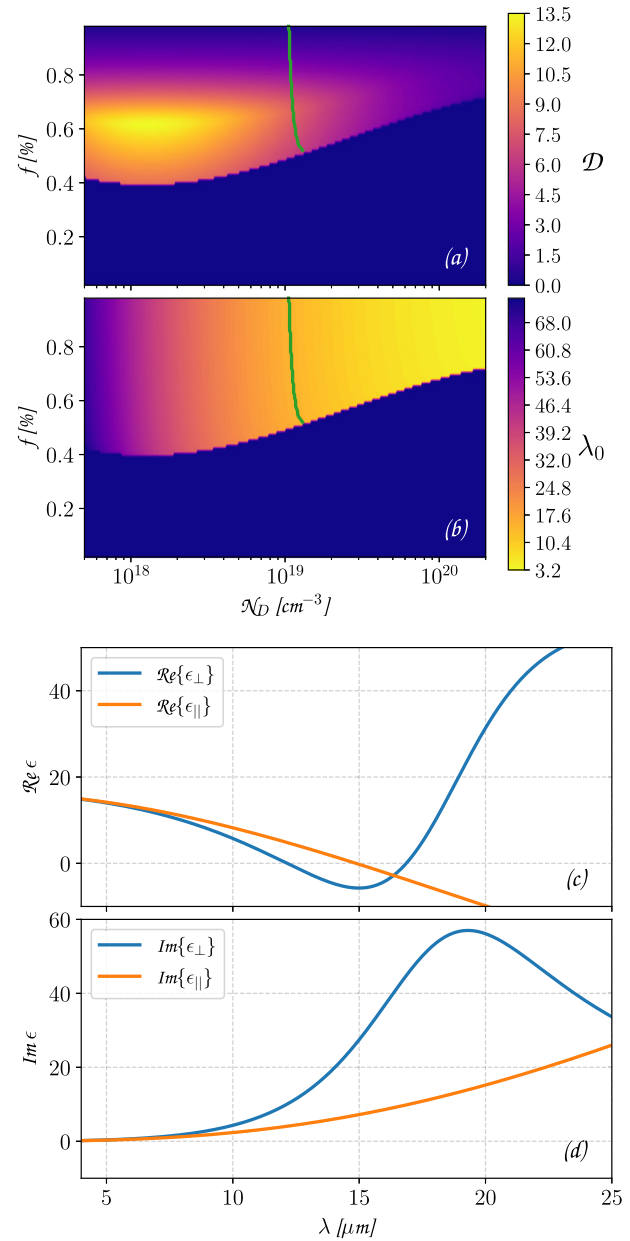


FIGURE 1 | (a) Depth D of the HMM region (see text) as a function of the active doping density N_D and filling factor f ; D is set to zero outside the HMM region. The area on the right side of the green curve corresponds to $\lambda_0 < 15\text{ }\mu\text{m}$; (b) λ_0 as a function of N_D and f . Real (c) and imaginary (d) part of $\epsilon_{||}$ and $\epsilon_{\perp}$ for $f = 0.65$ and $N_D = 1.8 \times 10^{19}\text{ cm}^{-3}$, as a function of λ . The yellow area evidence the spectral range of the HMM regime.

of $\sim 20\%$. Within this interval $\text{Re}(\epsilon_{\perp})$ reaches the minimum value of ≈ -5.4 .

For comparison, the authors of Ref. [11] studying heterostructures based on alternating doped $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$ and intrinsic $\text{Al}_{0.48}\text{In}_{0.52}\text{As}$ layers with hyperbolic behavior in a comparable spectral range, estimate $\text{Re}(\epsilon_{\perp})$ minima of about -60 . In this case the $|\text{Re}(k_{\text{TM}})|/|\text{Im}(k_{\text{TM}})|$ ratio calculated at the center of the HMM window is ~ 10 , a value approximately one order of magnitude larger than the one expected for the design of Figure 1c. This is due to the impact on the plasma frequency of the lighter effective mass of $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$ ($m_{\text{eff}} \approx 0.041m_0$). It

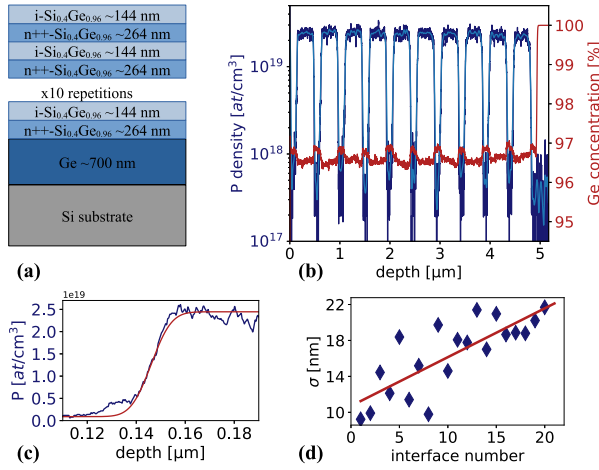


FIGURE 2 | (a) Sketch of the nominal structure of the sample. (b) SIMS measurements of the grown sample, showing a doping concentration of $2.42 \times 10^{19} \text{ cm}^{-3}$. The thicknesses of doped and undoped layers are $330 \pm 3 \text{ nm}$ and $150 \pm 4 \text{ nm}$, respectively. While they are roughly 20% above target, the filling factor is close to the nominal value. Ge content is also shown. (c) Detail of the topmost interface. The error function fit is also shown. (d) Values of σ for each interface and their linear interpolation.

follows that the dopant concentration required to target the same λ_0 is reduced by a factor of ~ 3 , leading to lower scattering rates ($\gamma \sim 1 \times 10^{13} \text{ s}^{-1}$), sharper resonances, and reduced losses.

The sample (sketched in Figure 2a) has been grown by means of UHV-CVD on a standard Silicon substrate, using SiH_4 and GeH_4 as precursor gases, targeting the peak D value indicated by our EMA modeling. The HMM is placed on top of a $\sim 1 \mu\text{m}$ Ge-on-Si virtual substrate, its P-doped layers realized by co-deposition of PH_3 . Since only a fraction of the dopants (estimated here to be 60%–70%) are electrically active at these doping levels [23, 24], we compensate by increasing the nominal doping density to $2.5 \times 10^{19} \text{ cm}^{-3}$. To accommodate such a high doping, we choose to employ a $\text{Si}_{0.04}\text{Ge}_{0.96}$ alloy instead of pure Ge, as the presence of a small percentage of Silicon facilitates the incorporation of Phosphorous without significantly altering the values of the material parameters employed in the modeling.

As for the thickness of each repetition, an upper bound is set by the target wavelength to ensure the validity of the EMA, which requires subwavelength dimensions. The lower boundary is given by the interdiffusion of phosphorus at the interfaces between doped and undoped regions. Figure 2c shows a detail of the topmost interface from Figure 2b. The SIMS profile can be accurately modeled with an error function, a fitting procedure yielding $\sigma \approx 9.2 \text{ nm}$. The values of σ for each of the interfaces shown by the SIMS profile in Figure 2b are reported in Figure 2d. The apparent increase of σ stems from the progressive loss of resolution typical of the technique as it etches deeper into the sample. Thus, we interpolate the values of σ obtaining an intercept of $\approx 11 \text{ nm}$, equivalent to a 10%–90% transition within $\sim 27 \text{ nm}$. Acknowledging such constraints, we choose a nominal thickness of $\sim 400 \text{ nm}$. As shown in Figure 2b, the grown structure is extremely regular and matches the nominal one quite precisely with respect to doping level, yielding $(2.42 \pm 0.04) \times 10^{19} \text{ cm}^{-3}$. The thicknesses of the grown layers are approximately 20%

above the nominal values ($330 \pm 3 \text{ nm}$ for the doped layers and $150 \pm 4 \text{ nm}$ for the undoped ones) yet the filling factor f is on point at $f = 0.68$.

3 | Reflectance Measurement and Simulation

To demonstrate the HMM character we estimate the spectral dependence of the parallel and perpendicular components of the dielectric tensor from grazing-incidence, polarization resolved reflectance experiments, performed with a Fourier transform infrared (FTIR) spectrometer where polarization was set using a modifiable linear polarizer. A gold mirror was used as reference. The reflection amplitudes of transverse-electric (r_{TE}) and transverse-magnetic waves (r_{TM}) impinging from air on a semi-infinite uniaxial material depend on $\epsilon_{\parallel}$ and $\epsilon_{\perp}$ according to [13, 14, 25]

$$r_{TE} = \frac{k_{\perp} - k_{TE}}{k_{\perp} + k_{TE}}, \quad (4)$$

$$r_{TM} = \frac{\epsilon_{\parallel} k_{\perp} - k_{TM}}{\epsilon_{\parallel} k_{\perp} + k_{TM}}, \quad (5)$$

$$k_{TE}^2 = \epsilon_{\parallel} (k_{\parallel}^2 + k_{\perp}^2) - k_{\parallel}^2, \quad (6)$$

$$k_{TM}^2 = \epsilon_{\parallel} (k_{\parallel}^2 + k_{\perp}^2) - \frac{\epsilon_{\parallel}}{\epsilon_{\perp}} k_{\parallel}^2 \quad (7)$$

where $k_{\parallel} = k \sin(\theta)$ and $k_{\perp} = k \cos(\theta)$ are the in plane and out-of-plane components of the incoming plane wave with wavevector $\vec{k}$, θ is the incidence angle, and k_{TE} and k_{TM} are the effective wavenumbers in the uniaxial material along the vertical axis for TE and TM polarization, respectively. We notice that in the above equations the TE reflectance is independent of the perpendicular permittivity as the electric field is always parallel to the sample surface. Consequently TE waves propagate in the material ordinarily, the negative refraction phenomenon occurring with TM radiation, since only in this latter case both $\epsilon_{\parallel}$ and $\epsilon_{\perp}$ are probed by the electric field.

Equations (4)–(5) cannot be used directly to fit experimental data due to the presence of multiple reflections which involve wave propagation in the buffer and wafer material. To account for this effect, we calculate the reflectance considering the complete multilayer stack, relying on the rigorous coupled wave approach [26]. In this framework, the metamaterial region is described in terms of alternating metallic and dielectric layers without relying on the EMA. As indicated by SIMS analysis, in our simulation we set a period of 480 nm , a doping density of $1.8 \times 10^{19} \text{ cm}^{-3}$, a fill factor of 0.65 and a virtual substrate thickness of $1.2 \mu\text{m}$.

The isotropic dielectric function of the metallic layers is modeled by Equation (1) while for the undoped layers we assume a real valued non dispersive permittivity $\epsilon_{\text{Si}_{0.05}\text{Ge}_{0.95}}$ of 16.36. The Si substrate and Ge VS regions are described using non-dispersive permittivities, setting $\epsilon_{\text{Si}} = 11.7$ and $\epsilon_{\text{Ge}} = 16.6$, respectively. For simplicity, perfectly sharp interfaces are assumed. This is justified by ad-hoc simulations showing that the influence of dopant

interdiffusion on reflectance can be effectively captured by a small reduction of the doping density. Due to its large thickness, the Si wafer is considered infinite, and the internal reflection at the Ge/Si interface is computed according to Fresnel's law.

Limiting to a single roundtrip pass, we compute the total reflected power by incoherent addition of directly reflected and first roundtrip powers as

$$R_{TE} = |r_{TE,0}|^2 + |r_{TE,1}|^2, \quad (8)$$

$$R_{TM} = |r_{TM,0}|^2 + |r_{TM,1}|^2. \quad (9)$$

where $r_{TE/TM,0}$ is the reflected amplitude through the full wave-simulated metamaterial and Ge virtual substrate, and $r_{TE/TM,1}$ is the first roundtrip reflected amplitude. This latter quantity is given by

$$r_{TE,1} = t_{TE,0}^2 T_{sub} \frac{k_{\perp} - k_{Si}}{k_{\perp} + k_{Si}}, \quad (10)$$

$$r_{TM,1} = t_{TM,0}^2 T_{sub} \frac{\epsilon_{Si} k_{\perp} - k_{Si}}{\epsilon_{Si} k_{\perp} + k_{Si}}, \quad (11)$$

$$k_{Si}^2 = \epsilon_{Si} (k_{\parallel}^2 + k_{\perp}^2) - k_{\parallel}^2. \quad (12)$$

where $t_{TE/TM,0}$ is the transmitted amplitude through the full wave-simulated metamaterial and virtual substrate, k_{Si} denotes the out-of-plane wavenumber in the Si substrate, and T_{sub} is a (phenomenological) factor describing the power losses due to propagation through the substrate, whose typical value for the investigated sample is about 0.6.

In our analysis, following Ref. [11], we focus on dichroic reflectance in order to suppress environmental fluctuations (polarization-resolved data are reported in Figures S3 and S4). Figure 3 shows the measured (a) and simulated (b) logarithm of the R_{TM}/R_{TE} ratio for our sample as a function of wavenumber and angle of incidence. In panel (a) the angle of incidence does not exceed 76 degrees due to geometrical constraints of our setup. The periodic oscillations in the reflectance are due to a Fabry-Pérot cavity effect, controlled by the combined thicknesses of the active HMM and Ge VS region and vanish in the long wavelength regime where the increase of the imaginary parts of the permittivities degrades the cavity finesse (see panel (d) in Figure 1) and the system enters the metallic regime. A singular peak in the reflectance ratio is observed at about $5 \times 10^{14} \text{ s}^{-1}$ ($3.8 \mu\text{m}$), arising from Bragg reflection associated with the periodicity of the multilayer. This feature lies beyond the validity range of the EMA, which cannot accurately describe diffraction effects due to the finite unit-cell thickness. We note that the rigorous coupled-wave approach employed here correctly captures this phenomenon.

Experiment and simulation show remarkable agreement, with less sharp and shallower features in the experimental data, an effect we attribute to some degree of angle spread featured by the incident beam. The accuracy of our modeling is also apparent from Figure 3c, where we show the dichroic transmittance at selected angles of incidence. Notice however that at high θ

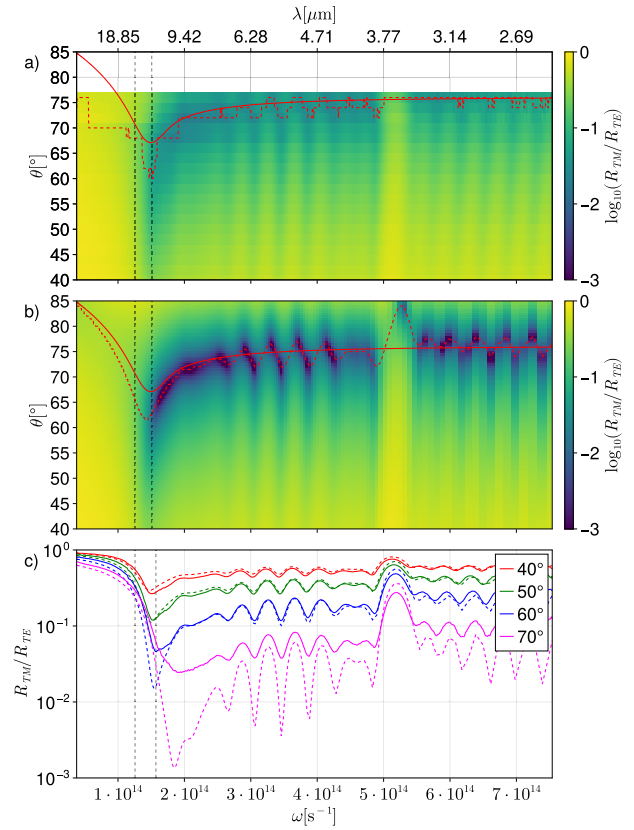


FIGURE 3 | Heatmaps of (a) measured and (b) simulated TM over TE reflectance ratios (log scale) as a function of wavenumber and angle of incidence. The red curves represent the pseudo-Brewster angle obtained minimizing the dichroic ratio as estimated in the EMA from Equations (4)–(5) (solid, both panels) and from the fully numerical simulation (dashed, central panel); the dashed curve in the top panel is obtained through minimization of the measured dichroic reflectance. Black vertical lines mark the zero crossings of real parts of the anisotropic permittivity at 13 and 15 μm . (c) Numerical (dashed) and measured (solid) dichroic reflectance at different incidence angles.

values the model overestimates the intensity of the wavenumber oscillations. This is likely due to the enlargement of the illumination spot at higher angles, which leads to a broader angular distribution of the incident beam, resulting in a more effective suppression of the Fabry-Pérot oscillations.

The topological transition from elliptic to type-I hyperbolic dispersion can be marked by identifying a discontinuity in the pseudo-Brewster angle $\theta_B(\lambda)$ [11], defined as the angle minimizing R_{TM} . Such discontinuity arises from the divergence of the $\epsilon_{\parallel}/\epsilon_{\perp}$ ratio when $\epsilon_{\perp}$ becomes negative, which induces a non-analytic behavior of the longitudinal TM wavevector k_{TM} and thus prevents the continuous evolution of the pseudo-Brewster condition. In Figure 3 we show the pseudo-Brewster angles corresponding to the minima of the measured (dashed curve, top panel) and calculated (dashed curves, central panel) dichroic reflectances. For comparison, the pseudo-Brewster angle calculated with the analytical model, i.e. Equations (4)–(5) which is valid in the limit of an infinitely thick metamaterial, is also reported (solid line, both panels). In the short wavelength range, consistent with the relative permittivities calculated according to

Equations (2) and (3) and reported in Figure 1, the HMM behaves as an anisotropic dielectric and exhibits a pronounced pseudo-Brewster angle which evolves smoothly. As the wavelength increases, the pseudo-Brewster angle undergoes a sharp variation when the epsilon-near-zero (ENZ) crossing of $\epsilon_{\perp}$, marking the transition from elliptic to type-I hyperbolic dispersion at approximately $\lambda_0 \simeq 13 \mu\text{m}$. At longer wavelengths, where $\text{Re}(\epsilon_{\perp}) < 0$ and the imaginary part of the permittivity becomes dominant, the multilayer enters a strongly dissipative, metal-like regime. In this wavelength region, incident radiation efficiently couples to lossy modes over a broad angular range, resulting in a uniformly low TM reflectance and in the disappearance of a well-defined pseudo-Brewster angle.

The observed behavior thus reflects a rapid crossover from impedance-controlled angular selectivity to absorption-dominated response, intrinsically linked to the hyperbolic character of the effective medium. In comparison with the data found in the literature (see for example Ref. [11]) R_{TM} over R_{TE} ratio reported in Figure 3 shows a rather weak discontinuity of the pseudo-Brewster angle. We argue that this is an intrinsic feature of the chosen material system. As previously mentioned, the effective electron mass in Ge is higher than in direct gap III-V compounds, such as GaAs, requiring higher doping concentrations to reach an equivalent ω_p and thus resulting in larger imaginary parts for the dielectric constants. This leads to a smoothing of the θ_B discontinuity and prevents the reemergence of a well defined pseudo-Brewster angle after the onset of the HMM regime. To substantiate this claim we performed a numerical experiment simulating the R spectra with a carrier mobility artificially enhanced by a factor of 10 to suppress optical losses. As apparent from Figure 4, this triggers a sharper transition and the reappearance of a well defined θ_B in the spectral region between the metallic and ordinary regime, similarly to what was reported in Ref. [11].

As an additional benchmark test, we have calculated reference heatmaps equivalent to Figure 3b) for a multilayer stack featuring either an undoped or an homogeneously doped layer at $1.9 \times 10^{19} \text{ cm}^{-3}$ with the same thickness of the HMM active region. In the undoped case, no dispersion is visible, and we just observe Fabry-Pérot oscillations. In the homogeneously doped case, the transition to a metal and the breakdown of Fabry-Pérot oscillations occur at higher frequencies as the effective doping is increased, and we only observe a slight decrease in the angle where the minimum dichroic ratio occurs (not shown).

To further substantiate the hyperbolic character of the metamaterial, we attempted to fit the real and imaginary parts of the anisotropic permittivity by comparing the measured reflectance data with numerical simulations performed using the EMA. For each subset of data with fixed wavelength, the anisotropic permittivity has been fitted independently. A similar procedure has been shown in Ref. [13], yet the peculiarities of our material system prevent the introduction of several approximations that greatly simplify the problem. Specifically, Ref. [13] ignores the imaginary parts of the permittivities and calculates the reflectivity disregarding interference effects due to multiple reflections. Within this simplified framework R_{TE} is decoupled from $\epsilon_{\perp}$. This allows to fit

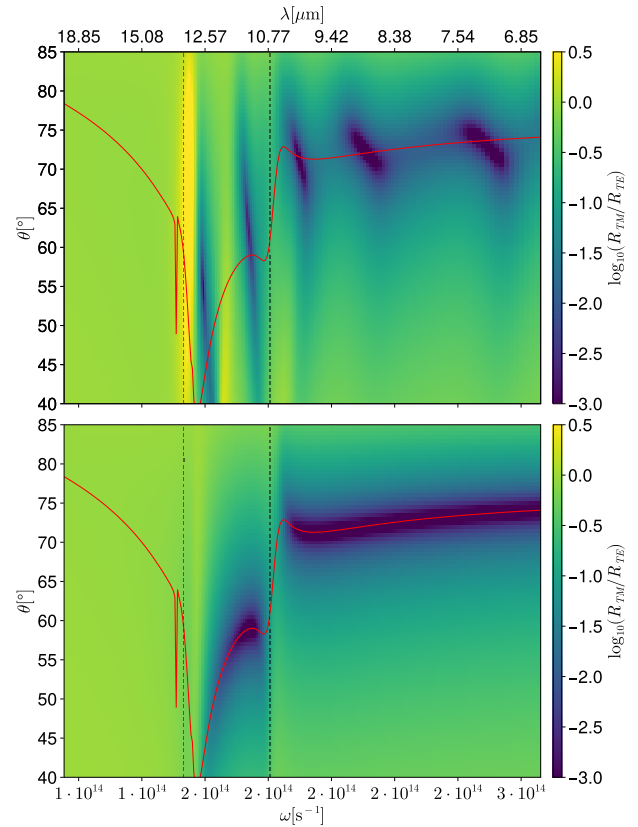


FIGURE 4 | Simulated ratio of TM and TE reflectance performed scaling the mobility by a factor of 10 for the full layer stack (top) and assuming an infinite thickness for the meta-material region (bottom). The red line represents the wavenumber-dependent pseudo-Brewster angle obtained through minimizing the effective-medium dichroic ratio. Black vertical lines mark the zero crossings of real parts of the anisotropic permittivities now found at 10.7 and 13.3 μm .

$\epsilon_{\parallel}$ from R_{TE} alone. Subsequently $\epsilon_{\perp}$ is obtained from R_{TM} taking $\epsilon_{\parallel}$ as a parameter. While the second assumption might be somewhat justified because its main drawback is the inability to account for Fabry-Pérot oscillations, the first one cannot be applied to the Ge material system because of the previously discussed large imaginary parts of the permittivity. To overcome these limitations, we rely on the rigorous coupled-wave approach, describing the HMM active multilayer region with an equivalent homogeneous layer characterized by the complex anisotropic permittivities resulting from the EMA. The complex $\epsilon_{\parallel}$ and $\epsilon_{\perp}$ are fit simultaneously, with the objective of matching experimental and simulated TE and TM reflectances. Using this approach, however, it turns out that the minimum of the fitting cost function is not well defined since it typically describes a circle in the complex plane for both the in-plane (Figure 5a) and out-of-plane permittivity (Figure 5b), thus hindering a robust evaluation of these physical quantities.

Aiming at better-defined cost function minimum, we performed an additional set of measurements at $\theta \simeq 70^\circ$ to acquire also phase information, using a scheme inspired by spectroscopic ellipsometry. For this purpose a linear polarizer is placed in the FTIR set-up before the mirrors controlling the grazing incidence on the sample. The polarized is set at an angle of 45° with respect

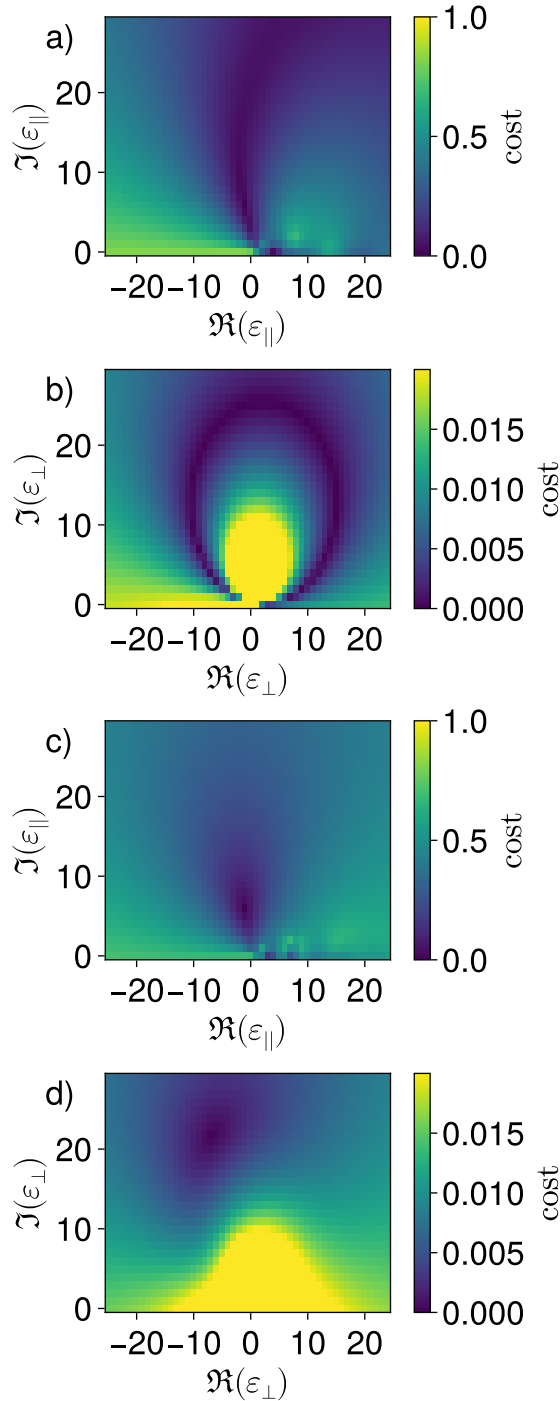


FIGURE 5 | Cost functions associated to (a) the in-plane permittivity $\epsilon_{||}$ and (b) the out-of-plane permittivity $\epsilon_{\perp}$ the fitting of the TE and TM reflectances calculated at $\lambda = 15\mu\text{m}$ and $\theta = 70^\circ$ using the EMA in conjunction with the rigorous coupled wave analysis. Panels (c) and (d) display improved cost functions obtained including the measured phase shift Δ in the fit.

to the plane of incidence, leading to a mixed TE/TM polarized beam. This results in elliptically polarized reflected waves, due to the different amplitudes and phases brought about by the TE and TM reflection amplitudes. To analyze this polarization state a second polarizer is placed before the detector. We recorded several measurements, rotating the analyzer in steps of 15 degrees. As

expected, the detected intensity at a fixed wavenumber oscillates sinusoidally as a function of the analyzer angle. From the three degrees of freedom of this sinusoid - amplitude, phase, and vertical offset - the TE and TM reflectance is computed again together with the phase shift Δ occurring between the TE and TM reflected waves, which has been evaluated taking into account the additional phase difference introduced by the gold parabolic mirrors present in the FTIR set-up between the sample and the two polarizers.

In our simulations we estimate Δ as

$$\cos \Delta = |r_{TM,0}r_{TE,0}| \cos \Delta_0 + |r_{TM,1}r_{TE,1}| \cos \Delta_1 \quad (13)$$

where

$$\tan \Delta_0 = \frac{\Im(r_{TM,0}/r_{TE,0})}{\Re(r_{TM,0}/r_{TE,0})} \quad (14)$$

$$\tan \Delta_1 = \frac{\Im(r_{TM,1}/r_{TE,1})}{\Re(r_{TM,1}/r_{TE,1})}. \quad (15)$$

Figure 5c,d illustrates how the additional information encoded in Δ improves the fitting procedure by providing better-defined minima in the cost function. Nevertheless, we find that the wavelength-by-wavelength fitting of the anisotropic permittivity results in a highly noisy spectral dependence for $\epsilon_{\perp}(\lambda)$. We therefore implemented a global fitting procedure based on Equations (1)–(7) and (8)–(12), in which the filling factor is estimated from SIMS measurements, while the active donor density in the doped layers (controlling both the plasma frequency and the damping factor), and the thickness of the virtual substrate are treated as free parameters. Since the modified FTIR setup hinders an accurate determination of the incidence angle, this quantity was also treated as a fitting parameter.

The fitting procedure yielded an active doping concentration of $1.62 \times 10^{19} \text{ cm}^{-3}$, corresponding to a plasma frequency $\omega_p = 6.57 \times 10^{14} \text{ s}^{-1}$ and a damping rate $\gamma = 5.08 \times 10^{13} \text{ s}^{-1}$, a virtual substrate thickness of $1.05 \mu\text{m}$, and an angle of incidence of 68.7° , in good agreement with the expected values thus providing confidence in the predictive capability of the global fitting approach for the estimation of the anisotropic permittivities. This conclusion is also supported by the good agreement between the experimental and simulated reflectance (Figure 6a) and phase difference Δ , shown in Figure 6a,b, respectively (the reduced modulation depth observed in the measurements is again attributed to a finite angular spread of the incident beam). The EMA permittivities obtained via Equation (1) with the fit value for the active doping concentration are reported in Figure 6c,d and indicate the achievement of a robust hyperbolic regime in the spectral range between ~ 13.5 and $\sim 15 \mu\text{m}$, matching well the corresponding interval shown in Figure 1c,d.

4 | Conclusion

We have experimentally demonstrated the emergence of a hyperbolic regime in a Ge-based multilayer metamaterial in the mid-infrared, between 12.5 and $15 \mu\text{m}$, through angle-resolved polarized reflectivity measurements. A key aspect of

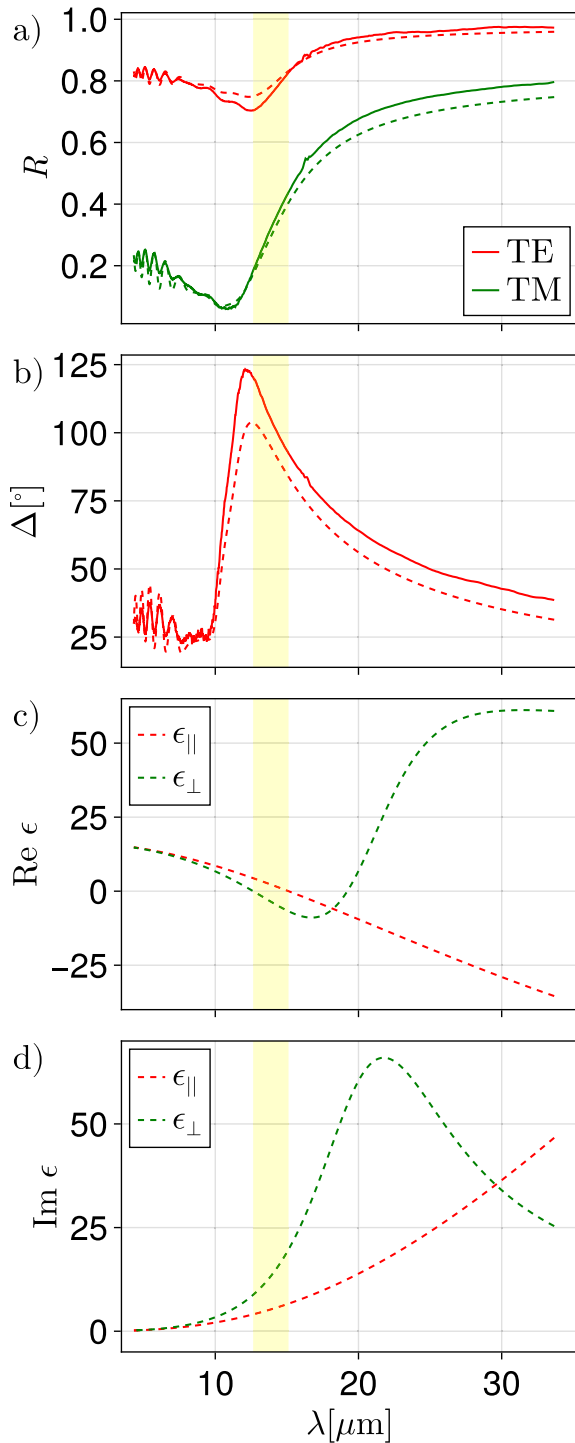


FIGURE 6 | Measured (solid) and fitted (dashed) (a) reflectance and (b) phase difference Δ . Real (c) and imaginary (d) parts of the parallel and perpendicular effective permittivity of the HMM sample as computed from the global fitting procedure (see text). The yellow area highlights the spectral range of the HMM regime.

this work is the use of a rigorous modeling framework that explicitly accounts for multiple reflections, finite-thickness effects, and intrinsic material losses, which are often neglected in simplified effective medium treatments. This approach reveals the central role of damping in this semiconductor platform, originating from the relatively large effective mass

associated with the L-valley of germanium, and shows how losses critically influence the accessible hyperbolic response. Furthermore, by complementing reflectance measurements with phase-sensitive information, we significantly improved the robustness and determinacy of the permittivity retrieval. This additional constraint proves essential for reliably extracting the anisotropic optical response of lossy hyperbolic metamaterials.

Beyond its compatibility with CMOS technology, a major advantage of this platform lies in the possibility of integrating hyperbolic functionality within existing Ge-based photonic architectures. In this context, the realization of a hyperbolic window in the mid-infrared is particularly relevant, as it enables applications in a spectral range of strong technological interest, including sensing, thermal photonics, and integrated infrared optics. In the short term, the most accessible applications of this Ge/SiGe HMM platform are expected to be those exploiting the local density of optical states and the strong anisotropic absorption rather than long-distance low-loss propagation. Promising examples include integrated mid-infrared evanescent-field sensors, where the HMM can enhance the interaction between guided fields and molecular vibrational resonances, as well as spectrally and angularly selective absorbers or thermal emitters. In addition, the planar geometry and compatibility with Ge-on-Si photonic architectures make this material system attractive for polarization- and angle-selective components in integrated infrared optical circuits. Looking forward, several promising research directions emerge. First, transmission experiments will be essential to directly probe negative refraction effects associated with the opposite signs of the real parts of the anisotropic permittivities. Second, the present approach can be extended to engineer hyperbolic dispersion over a broader infrared range by tuning the dopant concentration and filling factor within the same Ge-based heterostructure. We expect that the results presented here will stimulate further developments along these lines.

Acknowledgements

This work was partially funded by the Italian MUR under the PRIN 2022 PNRR project “Integrable THz Si-based Quantum Cascade Operation,” CUP I53D23006680001, funded by the European Union – NextGenerationEU. This work was partially funded by the German Federal Ministry of Research, Technology and Space (BMFTR) under grant 16ME0870. Part of this work has been carried out within the Joint Lab “Intelligent electro-optical sensing” established between IHP and Roma Tre University.

Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Endnotes

¹ Notice that minimizing the R_{TM} to R_{TE} ratio instead of R_{TM} yields essentially identical results; therefore the two will be used interchangeably throughout this section.

References

1. V. G. Veselago, "The Electrodynamics of Substances with Simultaneously Negative Values of ϵ and μ ," *Soviet Physics Uspekhi* 10, no. 4 (1968): 509–514, [10.1070/PUI968v010n04ABEH003699](https://doi.org/10.1070/PUI968v010n04ABEH003699).
2. J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, "Magnetism from Conductors and Enhanced Nonlinear Phenomena," *IEEE Transactions on Microwave Theory and Techniques* 47, no. 11 (1999): 2075–2084, <https://doi.org/10.1109/22.798002>.
3. D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, "Composite Medium with Simultaneously Negative Permeability and Permittivity," *Physical Review Letters* 84, no. 18 (2000): 4184–4187, <https://doi.org/10.1103/PhysRevLett.84.4184>.
4. R. A. Shelby, D. R. Smith, and S. Schultz, "Experimental Verification of a Negative Index of Refraction," *Science* 292, no. 5514 (2001): 77–79, <https://doi.org/10.1126/science.1058847>.
5. V. A. Podolskiy and E. E. Narimanov, "Strongly Anisotropic Waveguide as a Nonmagnetic Left-Handed System," *Physical Review B—Condensed Matter and Materials Physics* 71, no. 20 (2005): 201101.
6. Z. Jacob, I. I. Smolyaninov, and E. E. Narimanov, "Broadband Purcell Effect: Radiative Decay Engineering with Metamaterials," *Applied Physics Letters* 100, no. 18 (2012): 181105, <https://doi.org/10.1063/1.4710548>.
7. H. O. Moser, B. D. F. Casse, O. Wilhelm, and B. T. Saw, "Terahertz Response of a Microfabricated Rod-Split-Ring-Resonator Electromagnetic Metamaterial," *Physical Review Letters* 94, no. 6 (2005): 063901, <https://doi.org/10.1103/PhysRevLett.94.063901>.
8. P. Li, M. Lewin, A. V. Kretinin, et al., "Hyperbolic Phonon-Polaritons in Boron Nitride for Near-Field Optical Imaging and Focusing," *Nature Communications* 6 (2015): 7507, <https://doi.org/10.1038/ncomms8507>.
9. P. Shekhar, J. Atkinson, and Z. Jacob, "Hyperbolic Metamaterials: Fundamentals and Applications," *Nano Convergence* 1 (2014): 14, <https://doi.org/10.1186/s40580-014-0014-6>.
10. H. Lee, Z. Liu, Y. Xiong, C. Sun, and X. Zhang, "Development of Optical Hyperlens for Imaging Below the Diffraction Limit," *Optics Express* 15, no. 24 (Nov 2007): 15886–15891, <https://doi.org/10.1364/OE.15.015886>.
11. A. J. Hoffman, L. Alekseyev, S. S. Howard, et al., "Negative Refraction in Semiconductor Metamaterials," *Nature Materials* 6, no. 12 (2007): 946–950, <https://doi.org/10.1038/nmat2033>.
12. D. Wei, C. Harris, C. C. Bomberger, J. Zhang, J. Zide, and S. Law, "Single-Material Semiconductor Hyperbolic Metamaterials," *Optics Express* 24, no. 8 (2016): 8735–8745, <https://doi.org/10.1364/OE.24.008735>.
13. M. A. Noginov, Y. A. Barnakov, G. Zhu, T. Tumkur, H. Li, and E. E. Narimanov, "Bulk Photonic Metamaterial with Hyperbolic Dispersion," *Applied Physics Letters* 94, no. 15 (2009): 151105, <https://doi.org/10.1063/1.3115145>.
14. L. Hu and S. T. Chui, "Characteristics of Electromagnetic Wave Propagation in Uniaxially Anisotropic Left-Handed Materials," *Physical Review B* 66, no. 8 (2002): 085108, <https://doi.org/10.1103/PhysRevB.66.085108>.
15. K. Üstün and İ. H. Giden, "Comparative Optical Loss Analysis of Multilayer Type-I Hyperbolic Metamaterials," *Engineering Science and Technology, an International Journal* 68 (2025): 102087, <https://doi.org/10.1016/j.jestech.2025.102087>.
16. I. A. Fischer, M. Brehm, M. De Seta, et al., "On-Chip Infrared Photonics with Si-Ge-Heterostructures: What Is Next?," *APL Photonics* 7, no. 5 (2022): 050901, <https://doi.org/10.1063/5.0078608>.
17. R. Soref, "Mid-Infrared Photonics in Silicon and Germanium," *Nature Photonics* 4, no. 8 (2010): 495–497, <https://doi.org/10.1038/nphoton.2010.171>.
18. J. Frigerio, A. Ballabio, G. Isella, et al., "Tunability of the Dielectric Function of Heavily Doped Germanium Thin Films for Mid-Infrared Plasmonics," *Physical Review B* 94 (Aug 2016): 085202, <https://doi.org/10.1103/PhysRevB.94.085202>.
19. G. Dresselhaus, A. F. Kip, and C. Kittel, "Cyclotron Resonance of Electrons and Holes in Silicon and Germanium Crystals," *Physical Review* 98 (Apr 1955): 368–384, <https://doi.org/10.1103/PhysRev.98.368>.
20. E. Conwell and V. F. Weisskopf, "Theory of Impurity Scattering in Semiconductors," *Physical Review* 77 (Feb 1950): 388–390, <https://doi.org/10.1103/PhysRev.77.388>.
21. M. V. Fischetti and S. E. Laux, "Band Structure, Deformation Potentials, and Carrier Mobility in Strained Si, Ge, and SiGe Alloys," *Journal of Applied Physics* 80, no. 4 (1996): 2234–2252, <https://doi.org/10.1063/1.363052>.
22. C. Hilsum, "Simple Empirical Relationship Between Mobility and Carrier Concentration," *Electronics Letters* 10 (1974): 259–260, <https://doi.org/10.1049/el:19740205>.
23. M. R. Barget, M. Virgilio, G. Capellini, Y. Yamamoto, and T. Schroeder, "The Impact of Donors on Recombination Mechanisms in Heavily Doped Ge/Si Layers," *Journal of Applied Physics* 121, no. 24 (2017): 245701, <https://doi.org/10.1063/1.4986236>.
24. A. Vohra, I. Makkonen, G. Pourtois, et al., "Source/Drain Materials for Ge nMOS Devices: Phosphorus Activation in Epitaxial Si, Ge, $\text{Ge}_{1-x}\text{Sn}_x$ and $\text{Si}_y\text{Ge}_{1-x-y}\text{Sn}_x$," *ECS Journal of Solid State Science and Technology* 9, no. 4 (May 2020): 044010, <https://doi.org/10.1149/2162-8777/ab8d91>.
25. J. Lekner, *Theory of reflection* (Springer International Publishing, 2016).
26. J. Schlipf and I. A. Fischer, "Rigorous Coupled-Wave Analysis of a Multi-Layered Plasmonic Integrated Refractive Index Sensor," *Optics Express* 29, no. 22 (2021): 36201–36210.

Supporting Information

Additional supporting information can be found online in the Supporting Information section.

Supporting File: adom71760-sup-0001-SuppMat.pdf.